Chapter 22 Solutions

Problem 1:

A **+15 microC** charge is located **40 cm** from a **+3.0 microC** charge. The magnitude of the electrostatic force on the larger charge and on the smaller charge (**in N**) is, respectively,

Answer: 2.5, 2.5

Solution: The magnitiude of the electrostatic for is given by,

$$F = \frac{KQ_1Q_2}{r^2} = \frac{(8.99\times10^9 Nm^2/C^2)(15\mu C)(3\mu C)}{(40cm)^2} = 2.53N \,.$$

Remember the force on 1 due to 2 is **equal and opposite** to the force on 2 due to 1.

Problem 2:

Two point particles have charges $\mathbf{q_1}$ and $\mathbf{q_2}$ and are separated by a distance **d**. Particle $\mathbf{q_2}$ experiences an electrostatic force of **12 milliN** due to particle $\mathbf{q_1}$. If the charges of both particles are **doubled** and if the distance between them is **doubled**, what is the magnitude of the electrostatic force between them (**in milliN**)?

Answer: 12

Solution: The magnitude of the initial electristatic force is

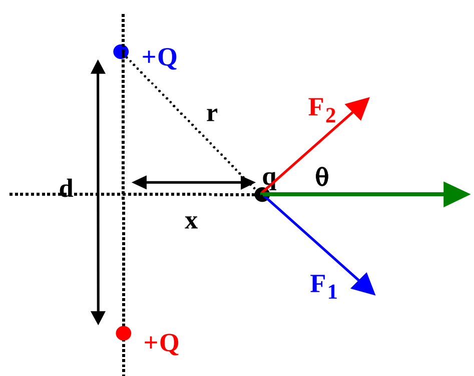
$$F_i = \frac{Kq_1q_2}{d^2} = 12mN \,.$$

The magnitude of the final electrostatic force is

$$F_f = \frac{K(2q_1)(2q_2)}{(2d)^2} = F_i = 12mN \,.$$

Problem 3:

Two identical point charges **+Q** are located on the y-axis at **y=+d/2** and **y=-d/2**, as shown in the **Figure**. A third charge **q** is placed on the x-axis. At what distance from the origin is the net force on **q** a **maximum**?



Answer: $d/(2\sqrt{2})$

Solution: The net force on **q** is the superposition of the forces from each of the two charges as follows:

$$\vec{F}_1 = \frac{KQq}{r^2}\left(\cos\theta\hat{x} - \sin\theta\hat{y}\right)$$

$$\vec{F}_2 = \frac{KQq}{r^2}\left(\cos\theta\hat{x} + \sin\theta\hat{y}\right)$$

$$\vec{F} = \vec{F}_1 + \vec{F}_2 = \frac{2KQq}{r^2}\cos\theta\hat{x}$$

$$= \frac{2KQqx}{(x^2 + (d/2)^2)^{3/2}}\hat{x}$$

where I used **$r^2 = x^2 + (d/2)^2$** and **$\cos\theta = x/r$.** To find the maximum we take the derivative of **F** and set it equal to zero as follows:

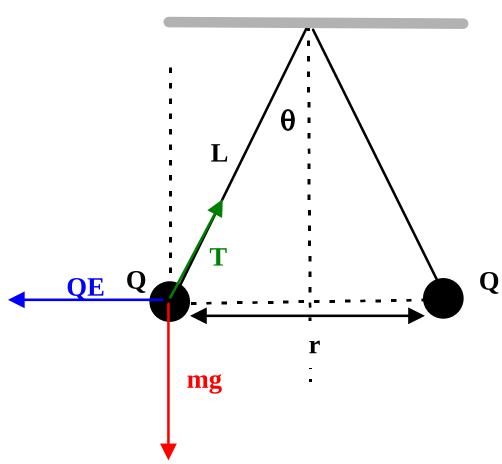
$$\frac{dF}{dx} = 2KQq\left(\frac{1}{(x^2+(d/2)^2)^{3/2}} - \frac{3x^2}{(x^2+(d/2)^2)^{5/2}}\right)$$

$$= \frac{2KQq\left((d/2)^2 - 2x^2\right)}{(x^2+(d/2)^2)^{5/2}} = 0$$

Solving for **x** gives $x = d/(2\sqrt{2})$.

Problem 4:

Two **2.0 gram** balls hang from lightweight insulating threads **50 cm** long from a common support point as shown in the **Figure**. When equal charges **Q** are place on each ball they are repelled, each making an angle of **10 degrees** with the vertical. What is the magnitude of **Q**, in microC?



Answer: **0.11**

Solution: Requiring the **x** and **y** components of the force to vanish yields

$$T\sin\theta = QE = \frac{KQ^2}{r^2}$$
$$T\cos\theta = mg$$

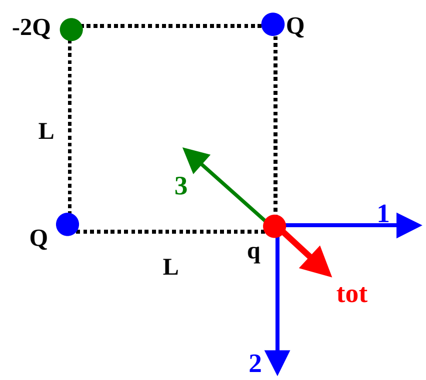
Eliminating the tension **T** and solving for $\mathbf{Q^2}$ gives

$$Q^2 = \frac{mgr^2\tan\theta}{K} = \frac{4mgL^2\sin^2\theta\tan\theta}{K}$$

and

$$Q = \sqrt{\frac{4(2\times10^{-3}kg)(9.8m/s^2)(0.5m)^2\sin^2 10^0\tan 10^0}{8.99\times10^9 Nm^2/C^2}} = 0.11\mu C\,.$$

Problem 5:



Four point charges **q**, **Q**, **-2Q**, and **Q** form a square with sides of length **L** as shown in the **Figure**. What is the magnitude of the resulting electrostatic force on the charge **q** due to the other three charges?

Answer: **$0.41KqQ/L^2$**

Solution: The electrostatic force on **q** is the **vector** superposition of the following three forces,

$$\vec{F}_1 = \frac{KqQ}{L^2}\hat{x}$$
$$\vec{F}_2 = -\frac{KqQ}{L^2}\hat{y}$$
$$\vec{F}_3 = \frac{2KqQ}{(\sqrt{2}L)^2}\left(-\frac{1}{\sqrt{2}}\hat{x} + \frac{1}{\sqrt{2}}\hat{y}\right)$$

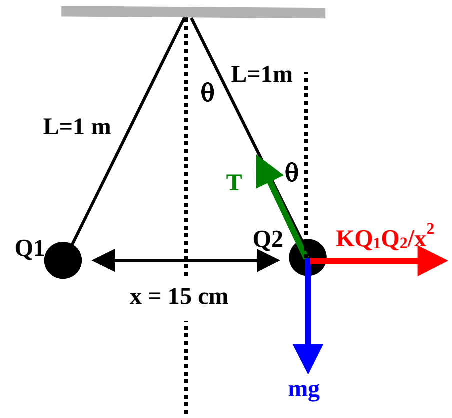
and the net force is thus,

$$\vec{F}_{tot} = \frac{KqQ}{L^2}\left(1 - \frac{1}{\sqrt{2}}\right)(\hat{x} - \hat{y})\,.$$

The magnitude is given by

$$\left|\vec{F}_{tot}\right| = \frac{KqQ}{L^2}\left(\sqrt{2}-1\right) = 0.41\frac{KqQ}{L^2}\,.$$

Problem 6:



Two **2.0 gram** charged balls hang from lightweight insulating threads **1 m** long from a common support point as shown in the **Figure**. If one of the balls has a charge of **0.01 microC** and if the balls are separated by a distance of **15 cm**, what is the charge on the other ball (**in microC**)?

Answer: **0.37**

Solution: The conditions for static equilibrium on ball Q2 is

$$T\sin\theta = \frac{KQ_1Q_2}{x^2}$$
$$T\cos\theta = mg$$

and dividing the 1st equation by the second yields

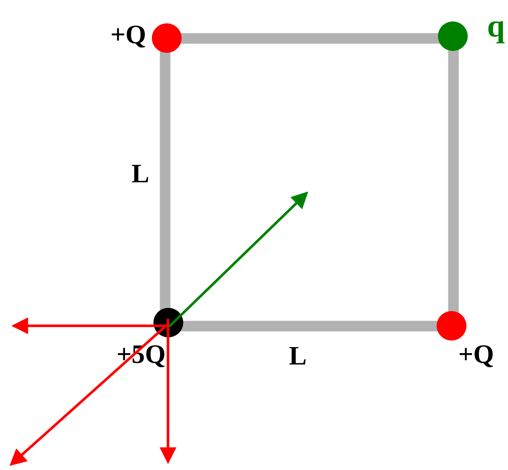
$$\tan\theta = \frac{KQ_1Q_2}{x^2mg}$$. Solving for the product of the two charges gives

$$Q_1Q_2 = \frac{x^3mg}{2LK} = \frac{(0.15m)^3(2\times10^{-3}kg)(9.8m/s^2)}{2(1m)(8.99\times10^9Nm^2/C^2)} = 3.68\times10^{-15}C^2$$

where I used **tanθ = x/(2L)**. Thus,

$$Q_2 = \frac{3.68\times10^{-15}C^2}{0.01\times10^{-6}C} = 0.368\mu C$$

Problem 7:



A charge **+Q** is fixed two diagonally opposite corners of a square with sides of length **L**. On another corner of the square a charge of **+5Q** if fixed as shown in the **Figure**. On the empty corner a charge is placed, such that there is no net electrostatic force acting on the **+5Q** charge. What charge is placed on the empty corner?

Answer: $-2\sqrt{2}Q$

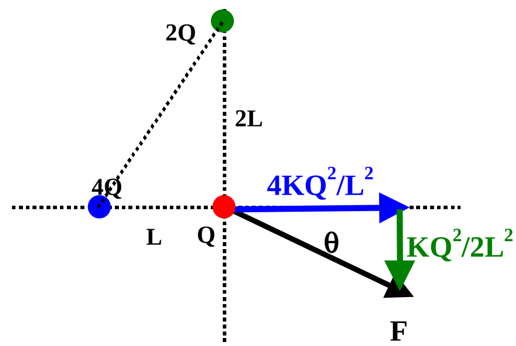
Solution: Setting the magnitude of the net for on the **+5Q** charge due to the two **+Q** charges equal to the magnitude of the force on the **+5Q** charge due to the charge **q** gives

$$\sqrt{2}\frac{KQ(5Q)}{L^2} = \frac{K|q|(5Q)}{\left(\sqrt{2}L\right)^2},$$

and solving for the magnitude of q, **|q|** (and noting that **q** must be **negative**) yields,

$$q = -2\sqrt{2}Q.$$

Problem 8:



Three point charges **Q**, **2Q**, and **4Q** form a triangle with sides of length **L** and **2L** as shown in the **Figure**. What angle does the resulting electrostatic force on the charge **Q** make with the positive x-axis (**in degrees**)?

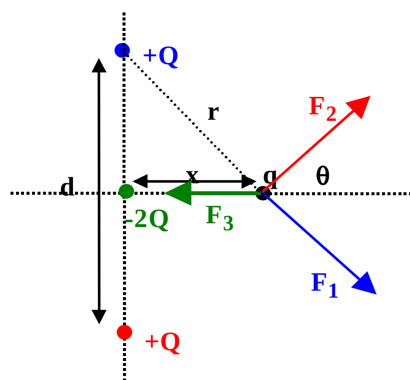
Answer: **7.1**

Solution: We see from the figure that

$$\tan\theta = \frac{2KQ^2/(2L)^2}{4KQ^2/L^2} = \frac{1}{8}$$

and hence $\boldsymbol{\theta = 7.1^o}$.

Problem 9:



Two identical point charges **+Q** are located on the y-axis at **y=+d/2** and **y=-d/2** and a third charge **-2Q** is located at the origin as shown in the **Figure**. The three charged +Q, +Q, and -2Q form an electrically neutral system called an ***electric quadrupole***. A charge q is placed on the x-axis a distance **x = d** from the quadrupole. At what is the net force on **q** due to the **quadrupole**?

Answer: $-0.57\dfrac{KQq}{d^2}\hat{x}$

Solution: The net force on **q** is the superposition of the following three vector forces:

$$\vec{F}_1 = \frac{KQq}{r^2}\left(\cos\,\theta\hat{x} - \sin\,\theta\hat{y}\right)$$
$$\vec{F}_2 = \frac{KQq}{r^2}\left(\cos\,\theta\hat{x} + \sin\,\theta\hat{y}\right)$$
$$\vec{F}_3 = -\frac{2\,KQq}{x^2}\,\hat{x}$$

where $\mathbf{r^2 = x^2 + (d/2)^2}$ and $\mathbf{\cos\theta = x/r}$. Adding the forces gives

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 2KQq\left(\frac{x}{(x^2+(d/2)^2)^{3/2}} - \frac{1}{x^2}\right)\hat{x}$$
$$= -\frac{2KQq}{x^2}\left(1 - \frac{1}{(1+(d/2x)^2)^{3/2}}\right)\hat{x}$$

which at **x = d** yields

$$\vec{F} = -\frac{2KQq}{d^2}\left(1 - \frac{1}{(1+(1/2)^2)^{3/2}}\right)\hat{x}$$
$$= -\frac{2KQq}{d^2}\left(1 - \frac{1}{(5/4)^{3/2}}\right)\hat{x} = -0.57\frac{KQq}{d^2}\,\hat{x}$$

Problem 10:

In the previous problem, what is the magnitude of force on the charge **q** when **x** becomes very large compared to the quadrupole separation **d**. (***Hint***: take the limit of the quadrupole force on **q** when **x** << **d**.)

Answer: $\mathbf{3KqQd^2/(4x^4)}$

Solution: From the previous problem we see that the magnitude of the quadrupole force is given by

$$\left|\vec{F}\right| = \frac{2KQq}{x^2}\left(1 - \frac{1}{(1+(d/2x)^2)^{3/2}}\right)$$
$$= \frac{2KQq}{x^2}\left(1-(1+\varepsilon)^{-3/2}\right) \approx \frac{3KQq\varepsilon}{x^2} = \frac{3KqQd^2}{4x^4}$$

where I used $\mathbf{(1+\varepsilon)^{-3/2} \sim 1 - 3\varepsilon/2}$ with $\mathbf{\varepsilon = (d/2x)^2}$.

Chapter 23 Solutions

Problem 1:

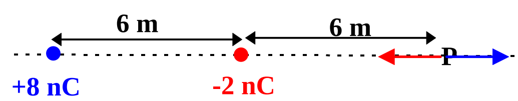
What is the magnitude (**in milliN**) and direction of the electrostatic force on a **-2.0 microC** charge in a uniform electric field given by $\vec{E} = (100\,N/C)\hat{x}$?

Answer: 0.2 in the -x direction

Solution: The electrostatic force is given by,

$$\vec{F} = q\vec{E} = (-2.0\mu C)(100\,N/C)\hat{x} = -0.2mN\hat{x}\,.$$

Problem 2:



Two point charges, **+8 nanoC** and **-2 nanoC** lie on the x-axis and are separated by **6 meters** as shown in the **Figure**. What is the magnitude of the electric field (**in N/C**) at a point **P** on the x-axis a distance of **6 meters** to the right of the negative charge?

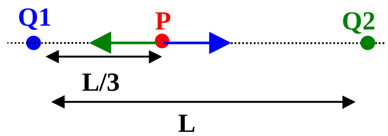
Answer: zero

Solution: The electric field at the point **P** is the superposition of the electric fields from each of the two charged as follows:

$$\vec{E}_1 = -\frac{KQ}{L^2}\hat{x}$$
$$\vec{E}_2 = \frac{K4Q}{(2L)^2}\hat{x} = \frac{KQ}{L^2}\hat{x}\,,$$
$$\vec{E}_{tot} = \vec{E}_1 + \vec{E}_2 = 0$$

where **Q = 2 nC** and **L = 6 m**.

Problem 3:



Two charges $\mathbf{Q_1}$ and $\mathbf{Q_2}$ are separated by distance **L** and lie on the x-axis with $\mathbf{Q_1}$ at the origin as shown in the **figure**. At a point **P** on the x-axis a distance **L/3** from $\mathbf{Q_1}$ the ***net*** electric field is zero. What is the ratio $\mathbf{Q_1/Q_2}$?

Answer: 0.25

Solution: In order for the net electric field to be zero it must be true that

$$\frac{KQ_1}{(L/3)^2} = \frac{KQ_2}{(2L/3)^2}$$, which implies that **Q_1/Q_2 = 1/4**.

Problem 4:

A particle with a charge to mass ratio of **0.1 C/kg** starts from rest in a uniform electric field with magnitude, **E = 10 N/C**. How far will the particle move (**in m**) in **2 seconds**?

Answer: 2

Solution: We know that

$$F = ma = qE$$ so that the acceleration is $$a = \left(\frac{q}{m}\right)E$$.

Thus, the distance traveled is

$$d = \frac{1}{2}at^2 = \frac{1}{2}\left(\frac{q}{m}\right)Et^2$$
$$= \frac{1}{2}(0.1C/kg)(10N/C)(2s)^2 = 2m$$

Problem 5:

A large insulating sheet carries a uniform surface charge of density **-1.2 microC/m^2**. A proton (with a mass of **1.67x10^{-27} kg**) is released from rest at perpendicular distance of **1.0 m** from the sheet. How much time (**in microsec**) elapses before the proton strikes the sheet?

Answer: 0.55

Solution: We know that **F = eE = Ma** so that **a = eE/M**. We also know that if the particle starts from rest then the distance traveled, **d = at^2/2**. Solving for t gives,

$$t = \sqrt{\frac{2d}{a}} = \sqrt{\frac{2dM}{eE}} = \sqrt{\frac{4dM\varepsilon_0}{e\sigma}}$$
$$= \sqrt{\frac{4(1m)(1.67\times10^{-27}kg)(8.85\times10^{-12}C^2/(Nm^2))}{(1.6\times10^{-19}C)(1.2\times10^{-6}C/m^2)}} = 0.55\mu s,$$

where I used **E = $\sigma/(2\varepsilon_0)$**.

Problem 6:

At a distance of **1 meter** from an isolated point charge the electric field strength is **100 N/C**. At what distance (**in m**) from the charge is the electric field strength equal to **50 N/C**?

Answer: 1.41

Solution: We know that $\mathbf{E_1=KQ/r_1^2}$ and $\mathbf{E_2=KQ/r_2^2}$ so that

$$\frac{E_1}{E_2}=\frac{r_2^2}{r_1^2} \quad \text{or} \quad r_2^2=r_1^2\frac{E_1}{E_2}=(1m)^2\frac{100N/C}{50N/C}=2m^2.$$

Hence, $r_2=\sqrt{2}m=1.41m$.

Problem 7:

The magnitude of the electric field **300 m** from a point charge **Q** is equal to **1,000 N/C**. What is the charge **Q** (**in C**)?

Answer: 0.01

Solution: We know that

$$E=\frac{KQ}{r^2}$$

and solving for **r** gives

$$Q=\frac{Er^2}{K}=\frac{(1{,}000N/C)(300m)^2}{8.99\times10^9 Nm^2/C^2}\approx 0.01C.$$

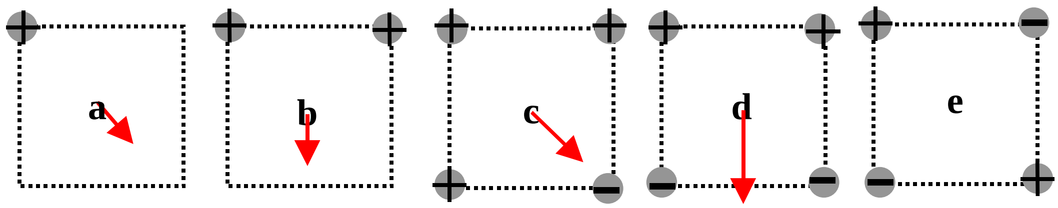
Problem 8:

A neon sign includes a long neon-filled glass tube with electrodes at each end across which an electric field of **20,000 N/C**. is placed. This large field accelerates free electrons which collide with and ionize a portion of the neon atoms which emit red light as they recombine. Assuming that some of the neon ions (mass $\mathbf{3.35x10^{-26}}$ **kg**) are singly ionized (i.e. have charge $\mathbf{e=1.6x10^{-19}}$ **C**) and are accelerated by the field, what is their acceleration ($\mathbf{m/s^2}$)?

Answer: $\mathbf{9.6x10^{10}}$

Solution: We know that **F = qE = ma**, so that

$$a = \frac{qE}{m} = \frac{(1.6\times10^{-19}C)(20{,}000N/C)}{3.35\times10^{-26}kg} = 9.6\times10^{10}m/s^2.$$



Problem 9:

Of the **five** charge configurations (a)-(e) shown in the **Figure** which one results in the maximum magnitude of the electric field at the center of the square? Each configuration consists of a square with either **+Q**, **-Q**, or no charge at each corner?

Answer: d

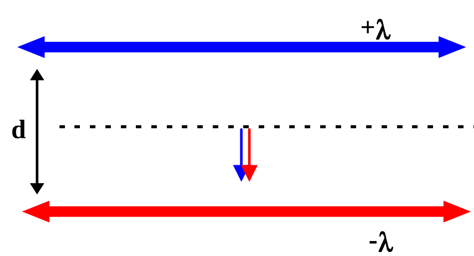
Solution: The magnitude of the electric field at the center of each square is given by

$$(a)\quad E = \frac{2KQ}{L^2}$$
$$(b)\quad E = 2\sqrt{2}\frac{KQ}{L^2}$$
$$(c)\quad E = \frac{4KQ}{L^2}$$
$$(d)\quad E = 4\sqrt{2}\frac{KQ}{L^2}$$
$$(e)\quad E = 0$$

where **L** is the length of the side of the square. The square with the largest E-field at the center is **(d)**.

Problem 10:

Two non-conducting infinite parallel rods are a distance **d** apart as shown in the **Figure**. If the rods have equal and opposite uniform charge density, λ, what is the magnitude of the electric field along a line that is midway between the two rods?

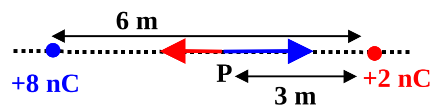


Answer: 8Kλ/d

Solution: The magnitude of the electric field from a single infinite rod is given by **E = 2Kλ/r**. Hence, the magnitude of the electric field along the line midway between the two rods is given by

$$E=\frac{2K\lambda}{d/2}+\frac{2K\lambda}{d/2}=\frac{8K\lambda}{d}.$$

Problem 11:



Two point charges, **+8 nanoC** and **+2 nanoC** are separated by **6 meters**. What is the magnitude of the electric field (**in N/C**) midway between them?

Answer: 6

Solution: The electric field at the point **P** is the superposition of the electric fields from each of the two charged as follows:

$$\vec{E}_1=\frac{K(8nC)}{(3m)^2}\hat{x}$$

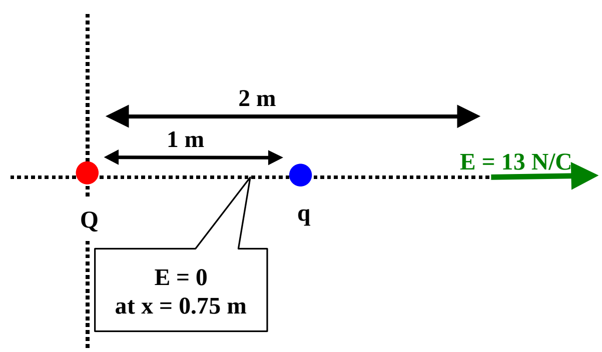
$$\vec{E}_2=-\frac{K(2nC)}{(3m)^2}\hat{x}$$

$$\vec{E}_{tot}=\vec{E}_1+\vec{E}_2=\frac{K(6nC)}{(3m)^2}\hat{x}$$

The magnitude of the **net** electric field is

$$|\vec{E}_{tot}|=\frac{K(6nC)}{(3m)^2}=\frac{(8.99\times10^9 Nm^2/C)(6\times10^{-9}C)}{9m^2}=6N/C.$$

Problem 12:



Two point charges, **Q** and **q**, lie on the **x-axis** and are separated by a distance of **1 meter** as shown in the **Figure**. The charge **Q** lies at the origin and the net electric field (on the x-axis) from the two charges is zero at the point **x = 0.75 m**. If the x-component of the electric field from the two charges, $\mathbf{E_x}$ **= 13 N/C**, at the point **x =2 m** (on the x-axis), what is **Q** (**in nanoC**)?

Answer: 4

Solution: The first condition (**net electric field zero at x = 0.75 m**) gives

$$\frac{KQ}{(0.75m)^2} = \frac{Kq}{(0.25m)^2} \quad \text{which implies } q = Q/9.$$

The second condition (**E = 13 N/C at x = 2 m**) gives

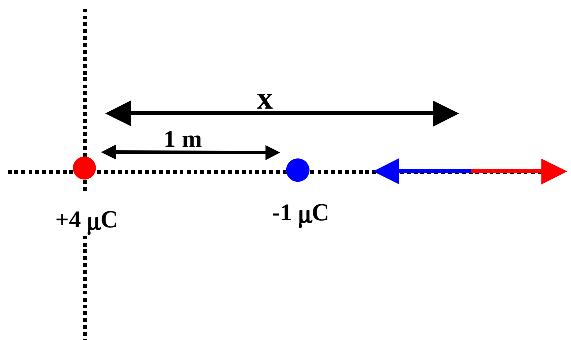
$$\frac{KQ}{(2m)^2} + \frac{Kq}{(1m)^2} = \frac{KQ}{(1m)^2}\left(\frac{1}{4}+\frac{1}{9}\right) = 13N/C$$

and solving for **Q** yields

$$Q = \frac{36(1m)^2(13N/C)}{13(8.99\times10^9 Nm^2/C^2)} \approx 4nC.$$

Problem 13:

Two point charges, **+4 microC** and **-1 microC**, lie on the x-axis and are separated by a distance of **1 meter** as shown in the **Figure**. If the +4 microC lies at the origin, at what point (or points) on the x-axis is the net electric field zero?



Answer: **x = 2.0 m**

Solution: The **first** condition that must be satisfied for the net electric field to be zero is that the magnitude of the E-field from the +4 mC charge must be equal to the magnitude of the E-field from the –1 mC charge. This implies that

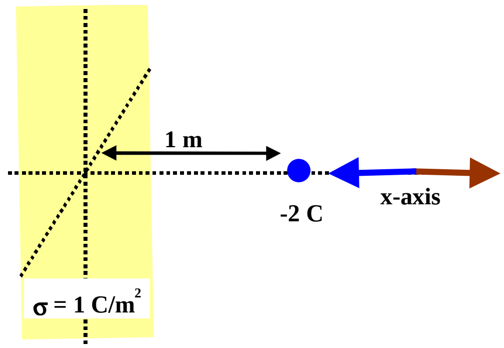
$$\frac{4}{x^2} = \frac{1}{(x-1m)^2}$$

Solving for the distance **x** yields

$$\frac{x}{(x-1m)} = \pm 2 \quad \text{or } \mathbf{x = 2\ m} \text{ and } \mathbf{x = 2/3\ m}.$$

The **second** condition that must be satisfied is that the vectors must point in opposite directions and is true only at the point **x = 2 m** (at x = 2/3 m they point in the same direction).

Problem 14:



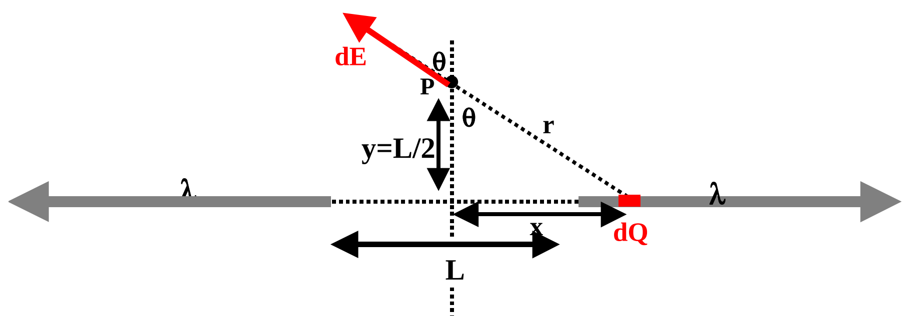
A **-2 C** point charge lies on the x-axis a distance of **1 meter** from an infinite non-conducting sheet of charge with a uniform surface charge density **σ = 1 C/m²** which lies in the yz-plane as shown in the **Figure**. At what point (or points) along the x-axis is the ***net*** electric field zero?

Answer: x=1.56 m

Solution: The **first** condition that must be satisfied for the net electric field to be zero is that the magnitude of the E-field from the infinite sheet must be equal to the magnitude of the E-field from the –2 C point charge. This implies that

$$\frac{|\sigma|}{2\varepsilon_0}=\frac{K|Q|}{(x-1m)^2} \quad \text{or} \quad (x-1m)^2=\frac{2K\varepsilon_0|Q|}{|\sigma|}=\frac{|Q|}{2\pi|\sigma|}$$

Solving for the distance **x** yields $x=(1\pm 0.564)m$. The **second** condition that must be satisfied is that the vectors must point in opposite directions and is true only at the point **x = 1.564 m** (at x = 0.436 m they point in the same direction).



Problem 15:

Two non-conducting semi-infinite rods lies along the x-axis as shown in the **Figure**. One rod lies from **x = L/2** to **x = infinity** and the other rod lies from **x = -L/2** to and **x = -infinity**. The rods have charge uniformly distributed along their length with a linear density **λ**. What is the magnitude of the electric field at a point **P** located on the y-axis at **y = L/2**?

Answer: 1.2Kλ/L

Solution: From the symmetry in the problem we see that the x-component of the electric field at the point **P** is zero and that the y-component of the electric field from the two rods is twice that from one of the rods as follows,

$$E_y = 2\int dE_y = 2\int \frac{KdQ}{r^2}\cos\theta$$

$$= 2K\lambda \int_{L/2}^{\infty} \frac{y}{(x^2+y^2)^{3/2}}dx = \frac{2K\lambda}{y}\left(1 - \frac{L/2}{\sqrt{y^2+(L/2)^2}}\right)$$

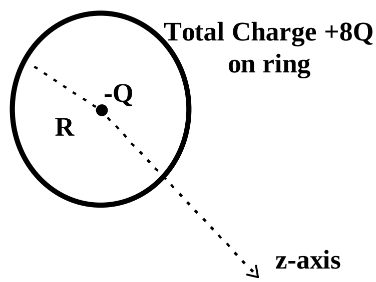
where I used $\mathbf{dQ = \lambda dx}$ and $\mathbf{\cos\theta = y/r}$ with $\mathbf{r^2 = x^2 + y^2}$. Plugging in $\mathbf{y = L/2}$ yields

$$E_y = \frac{2K\lambda}{L/2}\left(1 - \frac{L/2}{\sqrt{(L/2)^2+(L/2)^2}}\right)$$

$$= \frac{4K\lambda}{L}\left(1-\frac{1}{\sqrt{2}}\right) = 1.17\frac{K\lambda}{L}$$

Problem 16:

A positive charge **8Q** is distributed uniformly along a thin circular ring of radius **R**. If the ring has a charge **-Q** located at its center (see **Figure**), at what point , **z**, along the axis of the ring (***other than infinity***) is the net electric field zero?



Answer: $R/\sqrt{3}$

Solution: The magnitude of the net electric field on the z-axis will be zero provided

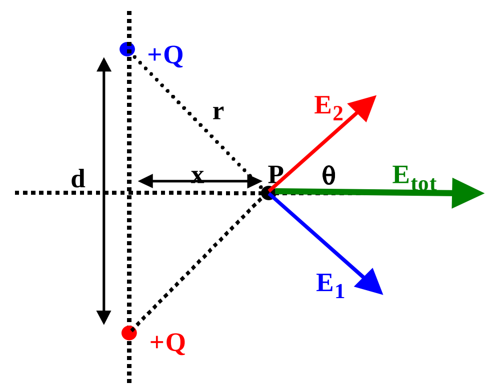
$$\frac{K(8Q)z}{(z^2+R^2)^{3/2}} = \frac{KQ}{z^2} \quad \text{or} \quad 8z^3 = \left(z^2+R^2\right)^{3/2}.$$

Taking the **2/3** power of both sided yields

$4z^2 = z^2 + R^2$ and solving for **z** gives, $z = R/\sqrt{3}$.

Problem 17:

Two identical point charges **+Q** are located on the y-axis at **y=+d/2** and **y=-d/2**, as shown in the **Figure**. What is the magnitude of the electric field on the x-axis a distance **x = d** from the origin?



Answer: 1.43KQ/d²

Solution: The net electric field at the point **P** is the superposition of the forces from each of the two charges as follows:

$$\vec{E}_1 = \frac{KQ}{r^2}\left(\cos\theta\hat{x} - \sin\theta\hat{y}\right)$$

$$\vec{E}_2 = \frac{KQ}{r^2}\left(\cos\theta\hat{x} + \sin\theta\hat{y}\right)$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{2KQ}{r^2}\cos\theta\hat{x}$$

$$= \frac{2KQx}{(x^2+(d/2)^2)^{3/2}}\hat{x}$$

where I used $r^2 = x^2 + (d/2)^2$ and $\cos\theta = x/r$. Setting **x = d** yields

$$|\vec{E}| = \frac{2}{(5/4)^{3/2}}\frac{KQ}{d^2} = 1.43\frac{KQ}{d^2}.$$

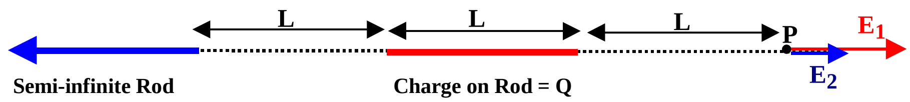
Problem 18:

In the previous problem, what is the magnitude of the electric field on the x-axis when **x** becomes very large compared to the separation **d** between the charges. (***Hint***: take the limit of the electric field when **x** >> **d**)?

Answer: 2KQ/x²

Solution: From the previous problem we see that

$$|\vec{E}| = \frac{2KQx}{(x^2+(d/2)^2)^{3/2}} = \frac{2KQ}{x^2(1+(d/2x)^2)^{3/2}} \rightarrow \frac{2KQ}{x^2}.$$



Problem 19:

A rod carrying a total charge **Q** uniformly distributed along its length **L** (constant λ) lies on the x-axis a distance **L** from a semi-infinite rod with the same uniform charge density **λ** as the finite rod as shown in the **Figure**. What is the magnitude of the ***net*** electric field at the point **P** on the x-axis a distance **L** from the end of finite rod?

Answer: **$5KQ/(6L^2)$**

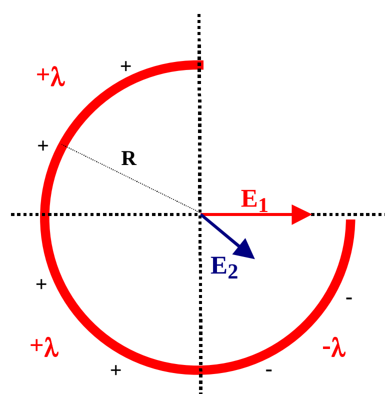
Solution: The net electric field at the point **P** is the superposition of the forces from each of the two rods (finite and semi-infinite) as follows:

$$\vec{E}_1 = \vec{E}_{rod} = \frac{KQ}{x(x+L)}\hat{x} = \frac{KQ}{2L^2}\hat{x}$$

$$\vec{E}_2 = \vec{E}_{semi} = \frac{K\lambda}{3L}\hat{x} = \frac{KQ}{3L^2}\hat{x}$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \left(\frac{1}{2}+\frac{1}{3}\right)\frac{KQ}{L^2}\hat{x} = \frac{5KQ}{6L^2}\hat{x}$$

where I used **λ = Q/L** and I used the fact that the point **P** was a distance **3L** from the end of the semi-infinite rod.

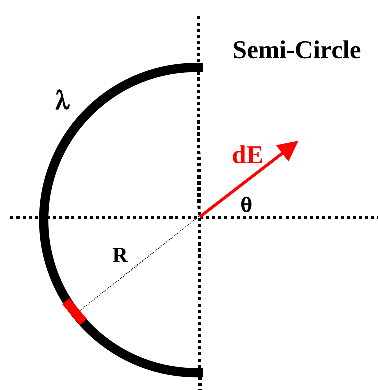


Problem 20:

A thin plastic rod is bent so that it makes a three quarters of a circle with radius **R** as shown in the **Figure**. If two quarters of the circle carry a uniform charge density **+λ** and one quarter of the circle carries a uniform charge density **-λ**, what is the magnitude of the electric field at the center?

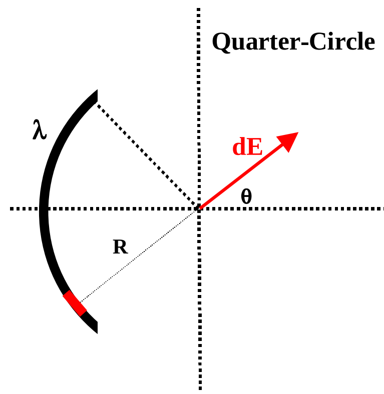
Answer: $\sqrt{10}K\lambda/R$

Solution: First I calculate the electric field at the center of a **semi-circle** as with charge density **λ** as follows:



$$E_x = \frac{K\lambda}{R}\int_{-\pi/2}^{\pi/2}\cos\theta d\theta = \frac{2K\lambda}{R},$$

where I have used **dE = KdQ/R^2** with **dQ = λds = λRdθ** and **dE$_x$ = dEcosθ**. Now I calculate the electric field at the center of a **quarter-circle** with charge density **λ** as follows:



$$E_x = \frac{K\lambda}{R}\int_{-\pi/4}^{\pi/4}\cos\theta d\theta = \frac{\sqrt{2}K\lambda}{R}.$$

The net electric field at the center of the three-quarter circle is the superposition of the fields from the semi-circle and the quarter-circle as follows:

$$\vec{E}_1 = \vec{E}_{semi} = \frac{2K\lambda}{R}\hat{x}$$

$$\vec{E}_2 = \vec{E}_{quarter} = \frac{\sqrt{2}K\lambda}{R}\left(\frac{1}{\sqrt{2}}\hat{x} - \frac{1}{\sqrt{2}}\hat{y}\right)$$

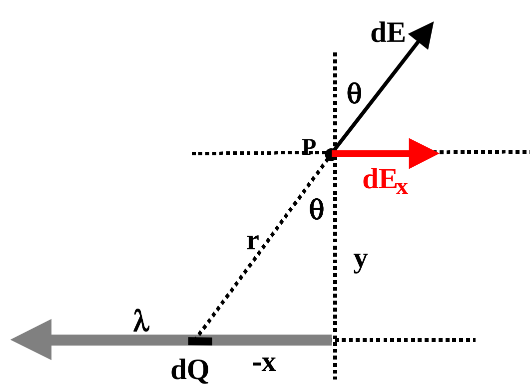
$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{K\lambda}{R}(3\hat{x} - 1\hat{y})$$

where I used $\sin 45^\circ = \cos 45^\circ = 1/\sqrt{2}$. The magnitude of the net electric field is thus

$$|\vec{E}| = \sqrt{9+1}\frac{K\lambda}{R} = \sqrt{10}\frac{K\lambda}{R}.$$

Problem 21:

A non-conducting semi-infinite rod lies along the x-axis as shown in the **Figure** (one end at x=0 and the other at x=-infinity). The rod has charge uniformly distributed along its length with a linear density **λ**. What is the x-component of the electric field, **E$_x$**, at a point **P** located a distance y above the end of the rod ?



Answer: Kλ/y

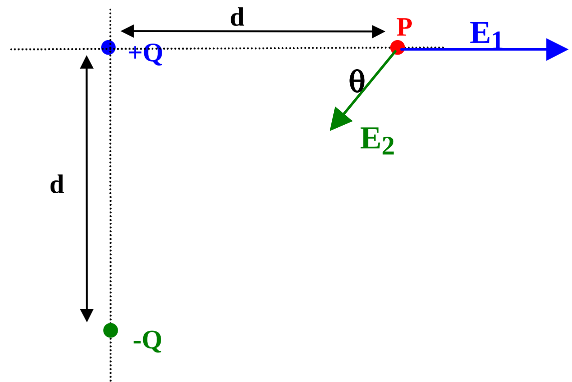
Solution: From the figure we see that

$$E_x = \int dE_x = \int \frac{KdQ}{r^2}\sin\theta$$

$$= -K\lambda \int_{-\infty}^{0} \frac{x}{(x^2+y^2)^{3/2}}dx = \frac{K\lambda}{y}$$

where I used **dQ = λdx** and **sinθ = -x/r** with $\mathbf{r^2 = x^2 + y^2}$.

Problem 22:



An electric dipole is placed on the y-axis with **+Q** at **y=0** and **-Q** at **y=-d**, as shown in the **figure**. What is the magnitude of the electric field at a point **P** located at **x=d** on the x-axis?

Answer: $\mathbf{0.74KQ/d^2}$

Solution: We superimpose the two E-fields as follows:

$$\vec{E}_1 = \frac{KQ}{d^2}\hat{x}$$

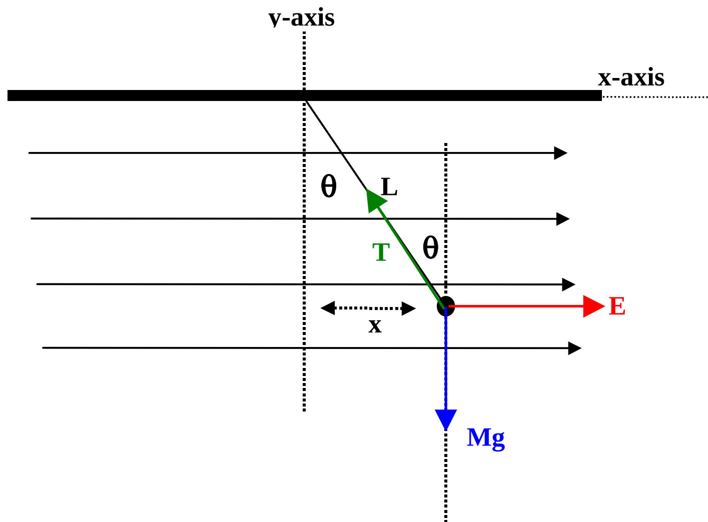
$$\vec{E}_2 = \frac{KQ}{2d^2}\left(-\frac{1}{\sqrt{2}}\hat{x} - \frac{1}{\sqrt{2}}\hat{y}\right)$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{KQ}{d^2}\left((1-\frac{1}{2\sqrt{2}})\hat{x} - \frac{1}{2\sqrt{2}}\hat{y}\right)$$

where I have used $\sin 45^\circ = \cos 45^\circ = 1/\sqrt{2}$. Solving for the magnitude yields,

$$E = \frac{KQ}{d^2}\sqrt{\left(1-\frac{1}{2\sqrt{2}}\right)^2 + \left(\frac{1}{2\sqrt{2}}\right)^2} = 0.737\frac{KQ}{d^2}.$$

Problem 23:



As shown in the **figure**, a ball of mass **M=2 kg** and charge **Q=3 C** is suspended on a string of negligible mass and length **L=1 m** in ***a non-uniform*** electric field $\vec{E}(x) = ax\hat{x}$, where **a=13.07 N/(Cm)** is a constant. If the ball hangs at a non-zero **θ** from the vertical, what is **θ**? (Hint: gravity pulls the ball down with acceleration $\mathbf{g=9.8\ m/s^2}$.)

Answer: 60^o

Solution: Requiring the **x** and **y** components of the force to vanish yields

$$T\sin\theta = QE$$

$$T\cos\theta = Mg.$$

Eliminating the tension **T** gives

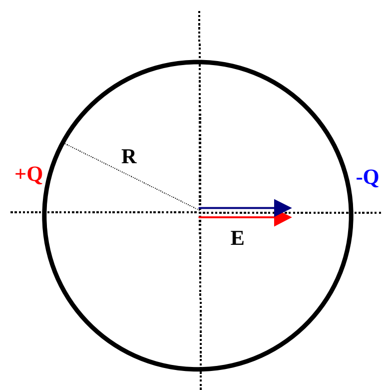
$$\frac{\sin\theta}{\cos\theta} = \frac{QE}{Mg} = \frac{Qax}{Mg} = \frac{QaL\sin\theta}{Mg},$$

and thus

$$\cos\theta = \frac{Mg}{QaL} = \frac{(2kg)(9.8m/s^2)}{(3C)(13.07N/(Cm))(1m)} = 0.5,$$

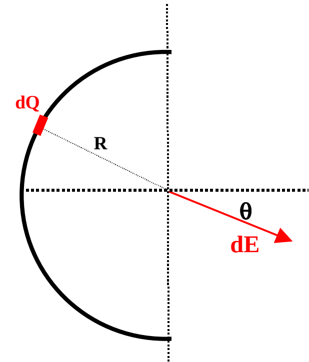
so that $\theta = 60^o$.

Problem 24:



Two semicircular uniform nonconducting loops, each of radius **R**, are charged oppositely to **+Q** and **-Q**, respectively. The loops are now joined end to end to form a circular loop as shown in the **figure**. What is the magnitude of the electric field at the center of the circular loop?

Answer: $4KQ/(\pi R^2)$

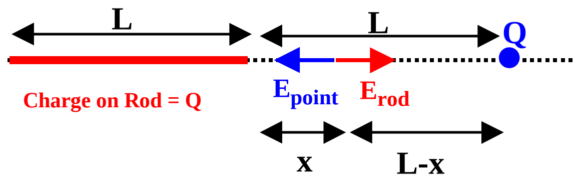


Solution: First I calculate the electric field at the center of a semi-circle as follows:

$$E_x = \frac{K\lambda}{R}\int_{-\pi/2}^{\pi/2}\cos\theta d\theta = \frac{2K\lambda}{R} = \frac{2KQ}{\pi R^2},$$

where I have used $\mathbf{dE = KdQ/R^2}$ with $\mathbf{dQ = \lambda ds = \lambda Rd\theta}$ and $\mathbf{dE_x = dE\cos\theta}$ and $\mathbf{Q = \pi R\lambda}$. Now for two semi-circles of opposite charge, the two electric fields add giving

$$E = E_1 + E_2 = \frac{2KQ}{\pi R^2} + \frac{2KQ}{\pi R^2} = \frac{4KQ}{\pi R^2}.$$



Problem 25:

A rod carrying a total charge **Q** uniformly distributed along its length **L** (constant λ) and a point charge **Q** both lie on the x-axis and are separated by a distance **L** as shown in the **Figure**. What is the magnitude of the ***net*** electric field at **P** on the x-axis a distance **x = L/3** from the end of the rod?

Answer: **zero**

Solution: We superimpose the two E-fields (**rod** and **point**) as follows:

$$\vec{E}_{rod} = \frac{KQ}{x(x+L)}\hat{x}$$

$$\vec{E}_{point} = -\frac{KQ}{(L-x)^2}\hat{x}$$

$$\vec{E} = \vec{E}_{rod} + \vec{E}_{point}$$

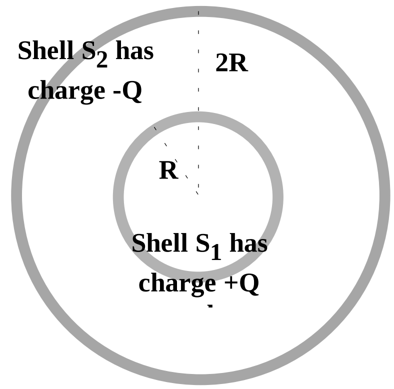
$$= KQ\left(\frac{1}{x(x+L)} - \frac{1}{(L-x)^2}\right)\hat{x}$$

At **x = L/3** we have

$$\vec{E} = \frac{KQ}{L^2}\left(\frac{9}{4} - \frac{9}{4}\right)\hat{x} = 0.$$

Chapter 24 Solutions

Problem 1:



Consider a spherical conducting shell $\mathbf{S_1}$ of radius **R** on which charge **+Q** is placed. Without touching or disturbing it, this shell is now surrounded concentrically by a similar shell $\mathbf{S_2}$ of radius **2R** on which charge **-Q** is placed (see Figure). What is the magnitude of the electric field in the region between the two shells (**R <r < 2R**)?

Answer: **KQ/r^2**

Solution: Gauss' Law tells us that

$$\Phi_E = \oint_{Surface} \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}.$$ If we choose our Gaussian surface to be a sphere with radius **r** such that **R < r < 2R** then we get $\mathbf{E(r)4\pi r^2 = Q/\varepsilon_0}$ and solving for **E** gives $\mathbf{E = KQ/r^2}$.

Problem 2:

In the previous problem, what is the electric field inside shell $\mathbf{S_1}$ (r < R)?

Answer: **zero**

Solution: Gauss' Law tells us that

$$\Phi_E = \oint_{Surface} \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}.$$ If we choose our Gaussian surface to be a sphere with radius **r** such that **r < R** then we get $\mathbf{E(r)4\pi r^2 = 0}$, since $Q_{enclosed} = 0$**,** and solving for **E** gives **E = 0**.

Problem 3:

A solid insulating sphere of radius **R** has charge distributed uniformly throughout its volume. What fraction of the sphere's total charge is located within the region **r < R/2**?

Answer: **1/8**

Solution: The amount of charge, **Q(r)**, within the radius **r < R** is given by

$$Q(r) = \int_0^r \rho dV = \int_0^r \rho 4\pi r^2 dr = \frac{4}{3}\pi r^3 \rho ,$$

where ρ is the volume charge density and **dV = 4πr²dr**. Thus,

$$\frac{Q(r)}{Q} = \left(\frac{r}{R}\right)^3 ,$$

where **Q = 4πR³ρ/3** is the sphere's total charge. For **r = R/2**, **(r/R)³ = 1/8**.

Problem 4:

A solid insulating sphere of radius **R** has a ***non-uniform*** volume charge distribution given by **ρ(r) = ar**, where **a** is a constant. What is the total charge **Q** of the insulating sphere?

Answer: **πaR⁴**

Solution: The amount of charge, **Q**, within the sphere is given by

$$Q(r) = \int_0^R \rho(r) dV = \int_0^R (ar) 4\pi r^2 dr = 4\pi a \int_0^R r^3 dr = \pi a R^4 .$$

Problem 5:

In a certain region of space within a distribution of charge the electric field is given by $\vec{E}(r) = ar\hat{r}$. It points radially away from the origin and has a magnitude **E(r) = ar**, where **a = 150N/(Cm)**. How much electric charge (**in nanoC**) is located inside a shell with an inner radius of **0.5 meters** and an outer radius of **1.0 meters**?

Answer: **14.6**

Solution: Gauss' Law tells us that

$$Q_{enclosed} = \varepsilon_0 \int E \cdot dA$$

$$= \varepsilon_0 (4\pi r_2^2 E(r_2) - 4\pi r_1^2 E(r_1)) = 4\pi\varepsilon_0 (r_2^2 E(r_2) - r_1^2 E(r_1))$$

$$= \frac{a}{K}(r_2^3 - r_1^3) = \frac{(150N/(Cm))((1m)^3 - (0.5m)^3)}{8.99\times10^9 Nm^2/C^2} = 14.6nC$$

Problem 6:

In a certain region of space the electric field is given by $\vec{E}(r) = (a/r)\hat{r}$. It points radially away from the origin and has a magnitude **E(r) = a/r**, where **a = 90Nm/C**. How much electric charge (**in nanoC**) is located inside a sphere with radius **R = 0.5** meters?

Answer: 5

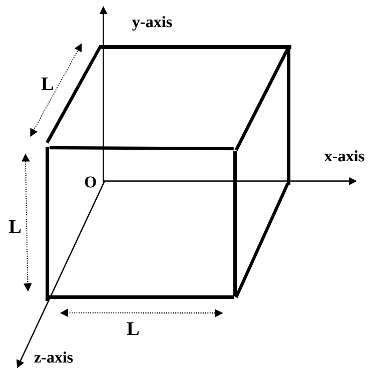
Solution: Gauss' Law tells us that

$$\Phi_E = \oint_{Surface} \vec{E}\cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}.$$

At the surface of a sphere of radius **R** this implies that $\mathbf{E(R)4\pi R^2 = Q/\varepsilon_0}$ and solving for **Q** and using $\mathbf{K=1/(4\pi\varepsilon_0)}$ gives,

$$Q = \frac{E(R)R^2}{K} = \frac{aR}{K} = \frac{(90Nm/C)(0.5m)}{8.99\times10^9 Nm^2/C^2} = 5nC.$$

Problem 7:



In a certain region of space within a distribution of charge the electric field is given by $\vec{E}(x) = ax\hat{x}$. It points in the **x direction** and has a magnitude $\mathbf{E_x(x) = ax}$, where **a = 150N/(Cm)**. How much electric charge (**in nanoC**) is located inside the cube with sides of length **L = 2 m** shown in the **Figure**?

Answer: 10.6

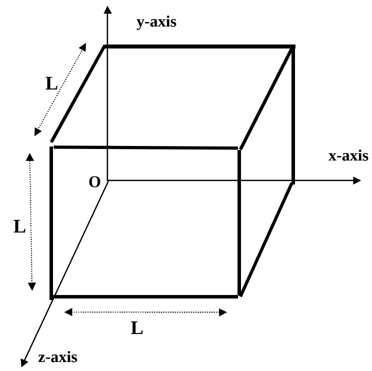
Solution: Gauss' Law tells us that

$$Q_{enclosed} = \varepsilon_0 \int E\cdot dA$$

$$= \varepsilon_0(E_x(L)L^2 - E_x(0)L^2) = \varepsilon_0 aL^3$$

$$= (8.85\times10^{-12}C^2/Nm^2)(150N/Cm)(2m)^3 = 10.6nC$$

Problem 8:



Consider a cube of sides **L=2 m**, as shown in the figure. Suppose that a ***non-uniform*** electric field is present and is given by $\vec{E}(x) = (a + bx)\hat{x}$, where **a=1 N/C** and **b=0.5 N/(Cm)**. What is the total ***net*** charge within the cube (**in picoC**)?

Answer: +35.4

Solution: Gauss' Law tells us that

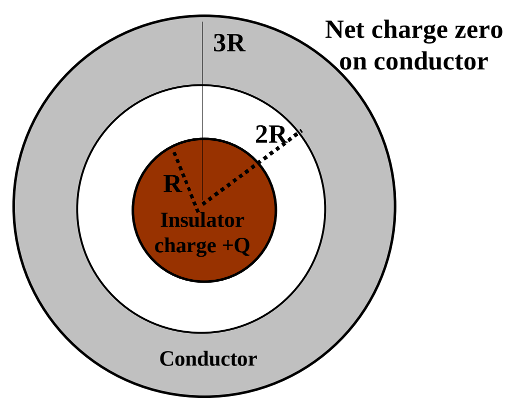
$$\Phi_E = \oint_{Surface} \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}.$$

At the surface of the cube

ϕ_E = (E(L)-E(0))L^2, and hence

$$Q = (E(L) - E(0))L^2\varepsilon_0 = (a + bL - a)L^2\varepsilon_0 = bL^3\varepsilon_0$$
$$= (0.5N/(Cm))(2m)^3(8.85\times10^{-12}C^2/(Nm)) = 35.4\,pC.$$

Problem 9:



A solid insulating sphere of radius **R** has a charge **+Q** distributed uniformly throughout its volume (the volume charge density **ρ** is constant). The insulating sphere is surrounded by a solid spherical conducting shell with inner radius **2R** and outer radius **3R** as shown in the **Figure**. The conductor is in static equilibrium and has a zero ***net*** charge. What is the magnitude of the electric field at the point **r = 5R/2** inside the conductor?

Answer: zero

Solution: The electric field inside a conductor in static equilibrium is zero!

Problem 10:

In the previous problem, what is the magnitude of the electric field at the point **r = 4R** outside the conductor?

Answer: KQ/(16R^2)

Solution: Gauss' Law tells us that

$$\Phi_E = \oint_{Surface} \vec{E}\cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$. For a Gaussian sphere of radius **r < 3R** this implies that **E(r)4πr^2 = Q/ε$_0$** and solving for **E(r)** and using **K=1/(4πε$_0$)** gives **E(r) = KQ/r^2** and plugging in **r = 4R** yields **E = KQ/(16R^2)**.

Problem 11:

A thunderstorm forms a mysterious spherical cloud with a radius of **20 m**. Measurements indicate that there is a uniform electric field at the surface of the cloud that has a magnitude of **1,000 N/C** and is directed radially inward toward the center of the cloud. What is the total ***net*** charge within the cloud (**in microC**)?

Answer: -44.5

Solution: Gauss' Law tells us that

$$\Phi_E = \oint_{Surface} \vec{E}\cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$. At the surface of the cloud

E(R)4πR^2 = Q/ε$_0$ and solving for **Q** gives,

$$Q = \frac{ER^2}{K} = \frac{(-1{,}000\,N/C)(20m)^2}{8.99\times10^9\,Nm^2/C^2} = -44.5\mu C.$$

Problem 12:

A **6.0 microC** charge is at the center of a cube **10 cm** on a side. What is the electric flux (**in Nm^2/C**) through one face of the cube? (***Hint***: think symmetry and don't do an integral.)

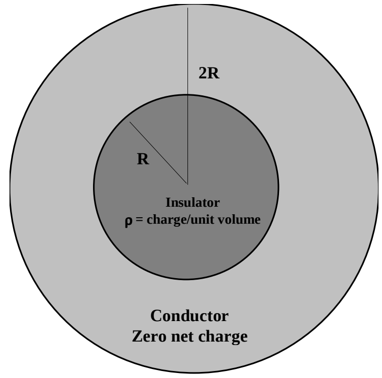
Answer: 1.13x10^5

Solution: The total flux through the cube is **ϕ_E = Q/ε$_0$** and symmetry tell us that the same amount of flux goes through each of the six faces of the cube. Hence,

$$\Phi_{face} = \frac{\Phi_E}{6} = \frac{Q_{enclosed}}{6\varepsilon_0} = \frac{6.0\times10^{-6}C}{6(8.85\times10^{-12}C^2/Nm^2)} = 1.13\times10^5\,Nm^2/C.$$

Problem 13:

A solid insulating sphere of radius **R** has charge distributed uniformly throughout its volume (the volume charge density ρ is constant). The insulating sphere is surrounded by a solid spherical conductor with inner radius **R** and outer radius **2R** as shown in the figure. The ***net*** charge on the conductor is zero. What is the magnitude of the electric field at the point **r=3R** outside the conductor?



Answer: **4πKRρ/27**

Solution: Gauss' Law tells us that

$$\Phi_E = \oint_{Surface} \vec{E}\cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}.$$

For a spherical surface outside the conductor **(r > 2R)** we have $\mathbf{E(r)4\pi r^2 = Q/\varepsilon_0}$ = $\mathbf{\rho 4\pi R^3/(3\varepsilon_0)}$ and solving for **E(r)** gives,

$$E(r) = \frac{4\pi K\rho R^3}{3r^2}.$$

For **r = 3R** we get **E = 4πKRρ/27**.

Problem 14:

In the previous problem, what is the magnitude of the electric field at the point **r=3R/2** inside the conductor?

Answer: **zero**

Solution: The electric field is zero inside a conductor in static equilibrium. This is our definition of a conductor!

Problem 15:

In the previous problem, what is the magnitude of the electric field at the point **r=R/2** inside the insulator?

Answer: **2πKRρ/3**

Solution: Gauss' Law tells us that

$$\Phi_E = \oint_{Surface} \vec{E}\cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}.$$

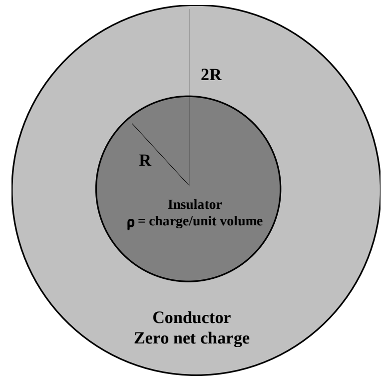
For a spherical surface inside the insulator **(r < R)** we have $\mathbf{E(r)4\pi r^2 = Q(r)/\varepsilon_0}$ = $\mathbf{\rho 4\pi r^3/(3\varepsilon_0)}$ and solving for **E(r)** gives,

$$E(r) = \frac{4}{3}\pi K\rho r\ .$$

For **r = R/2** we get **E = 2πKRρ/3**.

Problem 16:

A solid insulating sphere of radius **R** has a charge **+Q** distributed uniformly throughout its volume (the volume charge density **ρ** is constant). The insulating sphere is surrounded by a solid spherical conductor with inner radius **R** and outer radius **2R** as shown in the **Figure**. The conductor is in static equilibrium and has a ***net*** charge **+Q**. What is the magnitude of the electric field at the point **r = R/2** inside the insulating sphere?



Answer: **$KQ/(2R^2)$**

Solution: Gauss' Law tells us that

$$\Phi_E = \oint_{Surface} \vec{E}\cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}\ .$$

For a spherical surface inside the insulator with radius **r (r < R)** we have **$E(r)4\pi r^2 = Q(r)/\varepsilon_0$ = $\rho 4\pi r^3/(3\varepsilon_0)$** and solving for **E(r)** gives,

$$E(r) = \frac{4}{3}\pi K\rho r = \frac{KQr}{R^3}\ ,$$

where I used **$Q = 4\pi R^3\rho/3$**. For **r = R/2** we get **$E = KQ/(2R^2)$**.

Problem 17:

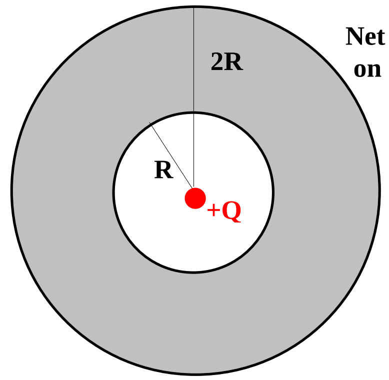
In the previous problem, what is the magnitude of the electric field at the point **r = 3R/2** inside the conductor?

Answer: **zero**

Solution: The electric field inside a conductor in static equilibrium is always **zero**.

Problem 18:

A point charge **+Q** is located at the center of a solid spherical conducting shell with inner radius of **R** and outer radius of **2R** as shown in the Figure. In addition, the conducting shell has a total ***net*** charge of **+Q**. How much charge is located on the outer surface (**r = 2R**) of the conducting shell?



Answer: **2Q**

Solution: We know that the net charge on the conductor is given by the sum of charge on the inner and outer surfaces, $\mathbf{Q_{net} = Q_{in} + Q_{out}}$. Gauss' Law tells us that

$$\Phi_E = \oint_{Surface} \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$

. If we choose our Gaussian surface to be a sphere with radius **r** such that R< **r < 2R** then **E = 0** (inside conducting material) which implies that $\mathbf{Q_{enclosed} = Q_{in} + Q = 0}$. Thus, $\mathbf{Q_{in} = -Q}$ and $\mathbf{Q_{out} = 2Q}$.

Problem 19:

In the previous problem, what is the magnitude of the electric field in the region **r < R** inside the hole in the conducting shell?

Answer: $\mathbf{KQ/r^2}$

Solution: Gauss' Law tells us that

$$\Phi_E = \oint_{Surface} \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$

. If we choose our Gaussian surface to be a sphere with radius **r** such that **r < R** then we get $\mathbf{E(r)4\pi r^2 = Q/\varepsilon_0}$, and solving for **E** gives $\mathbf{E = KQ/r^2}$.

Chapter 25 Solutions

Problem 1:

If the potential difference between points **A** and **B** is equal to V_A-V_B **= 3x10^4 Volts**, how much work (**in milliJoules**) must be done (*against the electric force*) to move a change particle (**Q = 3 microC**) from point **A** to point **B** without changing the kinetic energy of the particle?

Answer: -90

Solution: $W_{AB} = Q(V_B - V_A) = (3\text{x}10^{-6}\ \text{C})(3\text{x}10^4\ \text{V}) = 9\text{x}10^{-2}\ \text{J}$ = 90 microC. We know that the work done against the electric field must be negative since the particle will fall from high potential to low potential.

Problem 2:

Eight identical point charges, **Q**, are located at the corners of a cube with sides of length **L**. What is the electric potential at the center of the cube? (*Take V = 0 at infinity as the reference point.*)

Answer: 9.2KQ/L

Solution: The potential at the center of the cube is given by

$$V_{center} = \frac{8KQ}{d} = \frac{16}{\sqrt{3}}\frac{KQ}{L} = 9.24\frac{KQ}{L},$$

where I used $d = \sqrt{3}L/2$.

Problem 3:

What is the electric potential energy of the configuration of eight identical point charges, **Q**, at the corners of the cube with sides of length **L** in the previous problem?

Answer: 22.8KQ^2/L

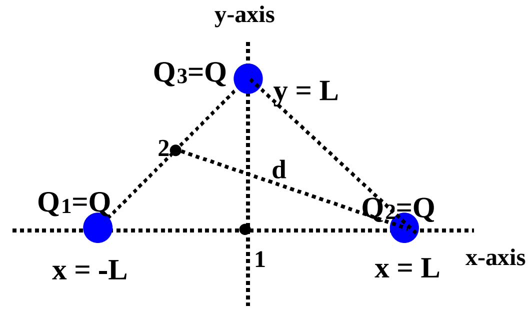
Solution: The total electric potential energy is given by

$$U_{cube} = \frac{1}{2}\sum_{i=1}^{8} Q_i V_i = \frac{8}{2} Q V_{corner} = 4Q\left(3 + \frac{3}{\sqrt{2}} + \frac{1}{\sqrt{3}}\right)\frac{KQ}{L} = 22.79\frac{KQ^2}{L},$$

where $\mathbf{V_{corner}}$ is the electric potential at the corner of the cube due to the other 7 charges.

Problem 4:

Two point charges $\mathbf{Q_1 = Q}$ and $\mathbf{Q_2 = Q}$ are on the x-axis at $\mathbf{x = +L}$ and $\mathbf{x = -L}$ and a third point charge $\mathbf{Q_3 = Q}$ is on the y-axis at the point $\mathbf{y = L}$ as shown in the **Figure**. What is the electric potential difference, $\mathbf{\Delta V = V_2 - V_1}$ between the point $\mathbf{P_2}$ at the midpoint of the line between $\mathbf{Q_1}$ and $\mathbf{Q_3}$ and the point $\mathbf{P_1}$ at the origin?



Answer: 0.5KQ/L

Solution: The electric potential at $\mathbf{P_1}$ is given by,

$$V_1 = 3\frac{KQ}{L}.$$

At $\mathbf{P_2}$ the electric potential is

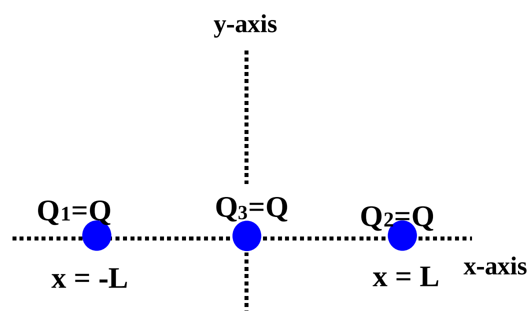
$$V_2 = \frac{2KQ}{\sqrt{2}L/2} + \frac{KQ}{d} = \left(\frac{4}{\sqrt{2}} + \frac{2}{\sqrt{10}}\right)\frac{KQ}{L}$$

where $d = \sqrt{(\sqrt{2}L/2)^2 + (\sqrt{2}L)^2} = \sqrt{10}L/2$. Thus,

$$V_2 - V_1 = \left(\frac{4}{\sqrt{2}} + \frac{2}{\sqrt{10}} - 3\right)\frac{KQ}{L} = 0.46\frac{KQ}{L}.$$

Problem 5:

Two point charges $\mathbf{Q_1 = Q}$ and $\mathbf{Q_2 = Q}$ are on the x-axis at $\mathbf{x = +L}$ and $\mathbf{x = -L}$ and a third point charge $\mathbf{Q_3 = Q}$ is on the y-axis at the point $\mathbf{y = L}$ as shown in the previous **Figure**. What is the electric potential energy of this charge configuration?



Answer: $\mathbf{1.9KQ^2/L}$

Solution: The electric potential energy of the charge configuration, $\mathbf{U_1}$, is given by,

$$U_1 = \frac{KQ^2}{2L} + \frac{2KQ^2}{\sqrt{2}L} = \left(\frac{1}{2} + \sqrt{2}\right)\frac{KQ^2}{L} = 1.91\frac{KQ^2}{L}.$$

Problem 6:

In the **problem 4**, how much work must be done (***against the electric force***) to move the charge $\mathbf{Q_3}$ from the point **y = L** on the y-axis to the origin resulting in the configuration shown in the **Figure**?

Answer: 0.6KQ^2/L

Solution: The potential potential energy of this new charge configuration, $\mathbf{U_2}$, is given by,

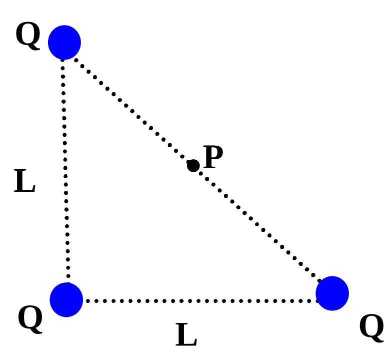
$$U_2 = \frac{KQ^2}{2L} + \frac{2KQ^2}{L} = \frac{5}{2}\frac{KQ^2}{L}.$$

and thus the work required is

$$W = U_2 - U_1 = \left(\frac{5}{2} - \frac{1}{2} - \sqrt{2}\right)\frac{KQ^2}{L} = \left(2 - \sqrt{2}\right)\frac{KQ^2}{L} = 0.59\frac{KQ^2}{L}.$$

Problem 7:

Three identical point charges, **Q**, are located at the corners of a right triangle with sides of length **L** as shown in the **Figure**. What is the electric potential at the point **P** that bisects the hypotenuse? (Take V = 0 at infinity as the reference point.)



Answer: 4.24KQ/L

Solution: The electric potential at the point **P** that bisects the hypotenuse is given by,

$$V = 3\frac{KQ}{r} = 3\sqrt{2}\frac{KQ}{L} = 4.24\frac{KQ}{L},$$

where I used $r = \sqrt{2}L/2 = L/\sqrt{2}$.

Problem 8:

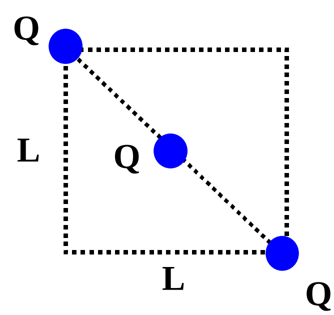
What is the electric potential energy of the configuration of three identical point charges, **Q**, located at the corners of a right triangle with sides of length **L** in the previous problem?

Answer: **2.71KQ^2/L**

Solution: The potential potential energy of the charge configuration, $\mathbf{U_1}$, is given by,

$$U_1 = 2\frac{KQ^2}{L} + \frac{KQ^2}{\sqrt{2}L}$$
$$= \left(2 + \frac{1}{\sqrt{2}}\right)\frac{KQ^2}{L} = 2.71\frac{KQ^2}{L}$$

Problem 9:



In **problem 7**, how much work is required to move the charge located at the 90^o angle of the equilateral triangle to the point **P** that bisects the hypotenuse?

Answer: **0.828KQ^2/L**

Solution: The potential potential energy of this new charge configuration, $\mathbf{U_2}$, is given by,

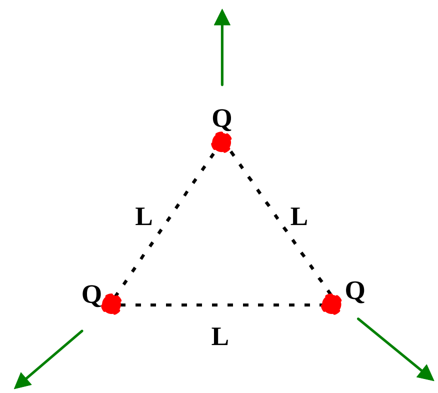
$$U_2 = \frac{KQ^2}{\sqrt{2}L} + 2\frac{KQ^2}{\sqrt{2}L/2}$$
$$= \left(\frac{1}{\sqrt{2}} + \frac{4}{\sqrt{2}}\right)\frac{KQ^2}{L} = \frac{5}{\sqrt{2}}\frac{KQ^2}{L}.$$

and thus the work required is

$$W = U_2 - U_1 = \left(\frac{5}{\sqrt{2}} - 2 - \frac{1}{\sqrt{2}}\right)\frac{KQ^2}{L}$$
$$= \left(\frac{4}{\sqrt{2}} - 2\right)\frac{KQ^2}{L} = 0.828\frac{KQ^2}{L}$$

Problem 10:

Three identical charged particles (**Q = 3.0 microC**) are placed at the vertices of an equilateral triangle with side of length **L= 1 m** and released simultaneously from rest as shown in the **Figure**. What is the kinetic energy (**in milliJ**) of one of the particles after the triangle has expanded to four times its initial area?



Answer: 40.5

Solution: Here we use energy conservation as follows:

$$E_i = 0 + 3\frac{KQ^2}{L}$$

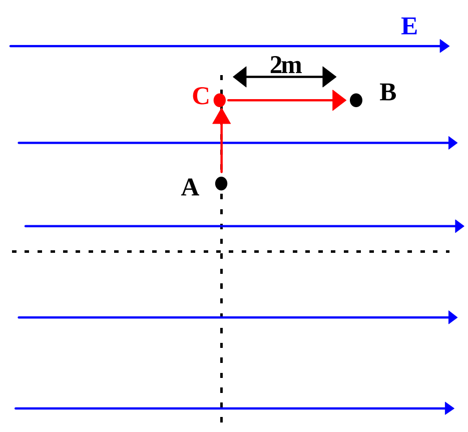
$$E_f = 3KE + 3\frac{KQ^2}{2L},$$

where **KE** is the kinetic energy of one of the particles and where $\mathbf{L_i = L}$ and $\mathbf{L_f = 2L}$ (since the area increased by factor of 4). Setting $\mathbf{E_i = E_f}$ yields

$$KE = \frac{KQ^2}{L}\left(1-\frac{1}{2}\right) = \frac{KQ^2}{2L} = \frac{(8.99\times10^9\,Nm^2/C^2)(3\times10^{-6}C)^2}{2(1m)} = 40.5mJ.$$

Problem 11:

A uniform electric field **E=10 V/m** points in the **x** direction as shown in the **Figure**. Point **A** has coordinates **(0,1m)** and point **B** has coordinated **(2m,2m)**, where points are labeled according to **P=(x,y)**. What is the potential difference $\mathbf{V_B - V_A}$ (**in Volts**)?



Answer: -20.0

Solution: The potential difference is equal to the following line integral,

$$V_B - V_A = -\int_A^B \vec{E}\cdot d\vec{l} = -\int_A^C \vec{E}\cdot d\vec{l} - \int_C^B \vec{E}\cdot d\vec{l},$$

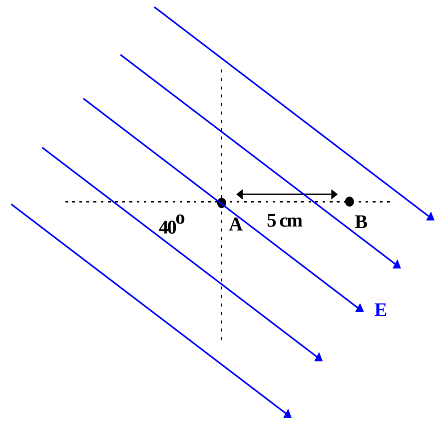
where I have chosen the path shown in the Figure. Evaluating the line integral along **AC** gives zero since **E** is perpendicular to the path and along the path **CB E** is parallel to the path so that,

$$\int_A^C \vec{E}\cdot d\vec{l} = 0$$

$$\int_C^B \vec{E}\cdot d\vec{l} = Ed = (10V/m)(2m) = 20V$$

and hence $\mathbf{V_B\text{-}V_A = -20V}$.

Problem 12:



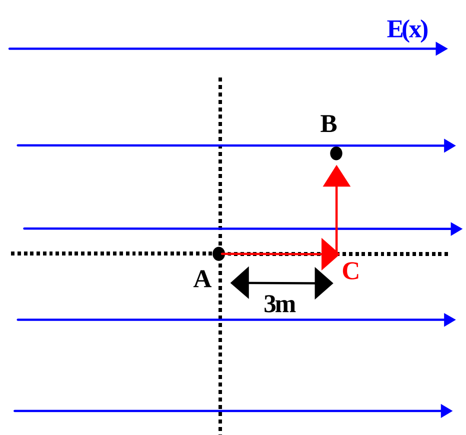
A uniform electric field **E=400 V/m** crosses the x-axis at a $\mathbf{40^o}$ angle as shown in the **Figure**. Point **B** lies on the x-axis **5 cm** to the right of the origin **A**. What is the potential difference $\mathbf{V_B\text{-}V_A}$ **(in Volts)**?

Answer: -15.3

Solution: We know that

$V_B - V_A = -\int_A^B \vec{E}\cdot d\vec{r}$ and hence $V_B\text{-}V_A$ = -E cosθ L = -(400 V/m) cos(40^o) (0.05 m) = -15.3 V.

Problem 13:



A non-uniform electric field is given by $\vec{E}(x) = ax^2\hat{x}$, where $\mathbf{a = 10\ V/m^3}$ as shown in the **Figure**. Point **B** lies at **x=y=3 m** and point **A** is at the origin (**x=y=0**). What is the potential difference $\mathbf{V_B\text{-}V_A}$ **(in Volts)**?

Answer: -90

Solution: The potential difference is equal to the following line integral,

$$V_B - V_A = -\int_A^B \vec{E}\cdot d\vec{l} = -\int_A^C \vec{E}\cdot d\vec{l} - \int_C^B \vec{E}\cdot d\vec{l},$$

where I have chosen the path shown in the Figure. Evaluating the line integral along **CB** gives zero since **E** is perpendicular to the path and along the path **AC E** is parallel to the path so that,

$$-\int_C^B \vec{E}\cdot d\vec{l} = 0$$

$$-\int_A^C \vec{E}\cdot d\vec{l} = -\int_0^3 ax^2 dx = -\frac{1}{3}a(3m)^3 = -90V$$

and hence $\mathbf{V_B - V_A = -90V}$.

Problem 14:

The potential along the x-axis is given by $\mathbf{V(x) = ax - bx^2}$, where $\mathbf{a = 2\ V/m}$ and $\mathbf{b = 1\ V/m^2}$. At what value(s) of **x** (**in m**) is the electric field equal to zero?

Answer: 1

Solution: The electric field is minus the derivative of the potential,

$$E_x = -\frac{dV}{dx} = -(a - 2bx) = 0.$$

Solving for **x** gives

$$x = \frac{a}{2b} = \frac{2V/m}{2(1V/m^2)} = 1m.$$

Problem 15:

The potential is given by $\mathbf{V(x,y) = a/(x^2+y^2)}$, where $\mathbf{a = 2\ Vm^2}$. What is the magnitude of the electric field (**V/m**) at the point, **P = (x,y) = (1m,1m)**?

Answer: 1.4

Solution: The three components of the electric field are given by,

$$E_x = -\frac{\partial V}{\partial x} = 2xa/(x^2+y^2)^2$$
$$E_y = -\frac{\partial V}{\partial y} = 2ya/(x^2+y^2)^2$$
$$E_z = -\frac{\partial V}{\partial z} = 0$$

and the magnitude of the electric field is

$$E = \sqrt{E_x^2+E_y^2+E_z^2} = \frac{2a}{(x^2+y^2)^{3/2}}.$$

At the origin **x = 1m** and **y = 1m** the magnitude of the electric field is

$$E = \frac{2(2Vm^2)}{(\sqrt{2}m)^3} = \sqrt{2}V/m = 1.41V/m.$$

Problem 16:

The potential is given by **V(x,y,z) = ax+by+cz^2**, where **a = 3 V/m**, **b = -4 V/m** ,and **c = 5 V/m^2**. What is the magnitude of the electric field (**in V/m**) at the origin, **P=(x,y,z)=(0,0,0)**?

Answer: 5

Solution: The three components of the electric field are given by,

$$E_x = -\frac{\partial V}{\partial x} = -a$$
$$E_y = -\frac{\partial V}{\partial y} = -b$$
$$E_z = -\frac{\partial V}{\partial z} = -2cz$$

and the magnitude of the electric field is

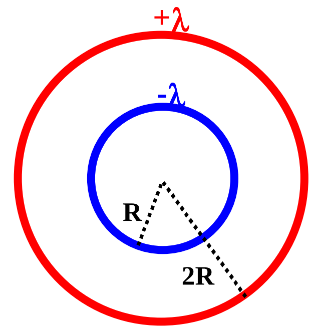
$$E = \sqrt{E_x^2+E_y^2+E_z^2} = \sqrt{a^2+b^2+4c^2z^2}.$$

At the origin z = 0 and the magnitude of the electric field is

$$E = \sqrt{a^2+b^2} = \sqrt{(3V/m)^2+(-4V/m)^2} = 5V/m.$$

Problem 17:

Two thin concentric circular plastic rods of radius **R** and **2R** have equal and opposite uniform charge densities **-λ** and **+λ**, respectively, distributed along their circumferences as shown in the **Figure**. What is the electric potential at the center of the two circles? (*Take V = 0 at infinity as the reference point.*)



Answer: zero

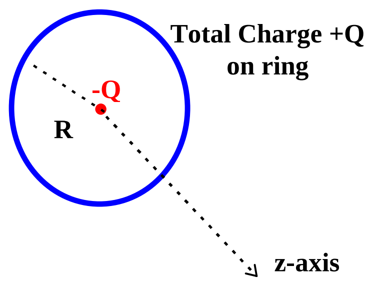
Solution: The electric potential at the center of the two circles is given by,

$$V = \frac{KQ_1}{r_1} + \frac{KQ_2}{r_2} = \frac{K2\pi r_1\lambda}{r_1} - \frac{K2\pi r_2\lambda}{r_2} = 2\pi K(\lambda - \lambda) = 0,$$

where the charge on a circular rod is **Q = 2πrλ**.

Problem 18:

A positive charge **Q** is distributed uniformly along a thin circular ring of radius **R**. A negatively charged particle (charge **-Q**) is at the center of the ring as shown in the **Figure**. What is the electric potential at a point on the z-axis of the ring a distance **R** from the center. (Take V=0 at infinity as the reference point.)



Answer: -0.293KQ/R

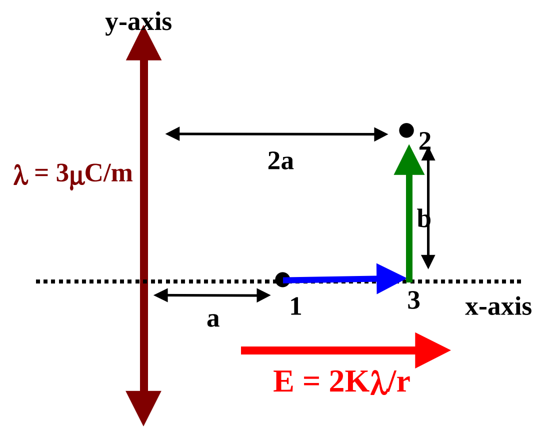
Solution: The net potential is the sum of the potential of the ring plus the potential of the point charge as follows:

$$V(z) = \frac{KQ}{\sqrt{z^2 + R^2}} - \frac{KQ}{z}$$ which at z =R becomes

$$V(R) = \frac{KQ}{R}\left(\frac{1}{\sqrt{2}} - 1\right) = -0.293\frac{KQ}{R}.$$

Problem 19:

An infinitely long thin rod lies along the y-axis and has a uniform linear charge density **λ = 3 microC/m** as shown in the **Figure**. If point $\mathbf{P_1 = (a,0)}$ and point $\mathbf{P_2 = (2a,b)}$, where **P = (x,y)**, what is the potential difference $\mathbf{\Delta V = V_2 - V_1}$ (**in kiloVolts**)?



Answer: -37.4

Solution: We know that the electric field from and infinite rod points away from the rod and has a magnitude given by **E(r) = 2Kλ/r**. The potential difference is equal to the following line integral,

$$\Delta V = V_2 - V_1 = -\int_1^2 \vec{E}\cdot d\vec{l} = -\int_1^3 \vec{E}\cdot d\vec{l} - \int_3^2 \vec{E}\cdot d\vec{l},$$

where I have chosen the path shown in the Figure. Evaluating the line integral along **32** gives zero since **E** is perpendicular to the path and along the path **13 E** is parallel to the path so that,

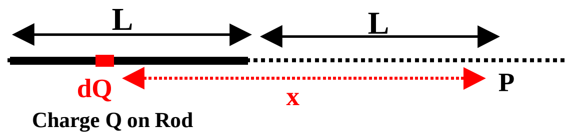
$$-\int_3^2 \vec{E}\cdot d\vec{l} = 0$$

$$-\int_1^3 \vec{E}\cdot d\vec{l} = -\int_a^{2a} \frac{2K\lambda}{x}dx = -2K\lambda\ln(2)$$

and hence

$$\Delta V = V_2 - V_1 = -2K\lambda\ln(2)$$

$$= -2(8.99\times10^9 Nm^2/C^2)(3\times10^{-6}C/m)\ln(2) = -37.4kV$$



Problem 20:

A positive charge **Q** is distributed uniformly along a thin rod of length **L** as shown in the **Figure**. What is the electric potential at a point **P** on the axis of the rod a distance **L** from the end of the rod? (Take V = 0 at infinity as the reference point.)

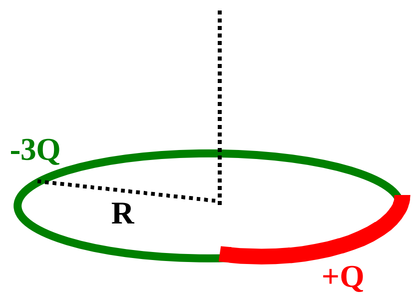
Answer: 0.693KQ/L

Solution: The electric potential at the point **P** is given by

$$V = \int_L^{2L} \frac{K\lambda dx}{x} = K\lambda \ln(2) = \ln(2)\frac{KQ}{L} = 0.693\frac{KQ}{L},$$

where I used **dQ = λdx**.

Problem 21:



A circular plastic rod of radius **R** has a positive charge **+Q** uniformly distributed along one-quarter of its circumference and a negative charge of **-3Q** uniformly distributed along the rest of the circumference as shown in the **Figure**. What is the electric potential at the center of the circle? (Take V = 0 at infinity as the reference point.)?

Answer: -2KQ/R

Solution: The electric potential at the center of the circle is given by,

$$V = \frac{KQ}{R} - \frac{3KQ}{R} = -2\frac{KQ}{R}.$$

Problem 22:

The electric field at the surface of a solid spherical conductor (radius **R = 20 cm**) points radially outward and has a magnitude of **10 V/m**. Calculate the electric potential (**in Volts**) at the center of the conducting sphere**. *(Take V = 0 at infinity as the reference point*.**)

Answer: 2

Solution: At the surface of the conductor we know that,

$$V_{surface} = \frac{KQ}{R} \quad E_{surface} = \frac{KQ}{R^2}.$$

Hence,

$$V_{center} = V_{surface} = E_{surface}R = (10V/m)(0.2m) = 2V.$$

Problem 23:

A solid insulating sphere of radius **R** has charge a total charge **Q** distributed uniformly throughout its volume (the volume charge density ρ is constant). If the electric potential is zero at infinity and **12 Volts** at the center of the sphere, what is the potential at the surface **r = R** (**in Volts**)?

Answer: 8

Solution: The electric field inside the insulator (r < R) is given by **E(r) = KQr/R^3** and the potental difference between the center and the surface is,

$$V_{surface} - V_{center} = -\int_0^R \vec{E}\cdot d\vec{r} = -\int_0^R \frac{KQr}{R^3}dr = -\frac{1}{2}\frac{KQ}{R}.$$

We know that **$V_{surface}$ = KQ/R** which means that

$$V_{center} = \frac{3}{2}\frac{KQ}{R} \quad \text{and} \quad V_{surface} = \frac{2}{3}V_{center} = 8V.$$

Chapter 26 Solutions

Problem 1:

What is the capacitance **C** of a solid spherical conducting shell with inner radius **R** and outer radius **2R**?

Answer: **2R/K**

Solution: The electric potential of the solid spherical conductor is given by

$$V = \frac{KQ}{r} = \frac{KQ}{2R},$$

where **r = 2R** is the surface of the conductor. Hence,

$$C = \frac{Q}{V} = \frac{2R}{K}.$$

Problem 2:

An energy of **9.0x10^{-3} Joules** is stored by a capacitor that has a potential difference of **85 Volts** across it. Determine the capacitance of the capacitor (**in microF**).

Answer: **2.5**

Solution: We know that

$$U = \frac{1}{2}C(\Delta V)^2,$$

and hence

$$C = \frac{2U}{(\Delta V)^2} = \frac{2(9\times 10^{-3}J)}{(85V)^2} = 2.5\mu F.$$

Problem 3:

Capacitor **#1** with capacitance **C** and initial charge **Q** is connected in parallel across an initially uncharged capacitor with capacitance **2C** (**#2**). After the system comes to equilibrium, what is the charge on capacitor **#1**?

Answer: **Q/3**

Solution: After the two capacitors are connected their charges must satisify

$$Q_1 + Q_2 = Q$$
$$\frac{Q_1}{C} = \frac{Q_2}{2C}$$

whre the first condition comes from charge conservation and the second comes from the fact that the potential drop across each capacitor is the same since they are connected in parallel. Solving for $\mathbf{Q_1}$ yields

$$Q_1 = \frac{Q}{3}.$$

Problem 4:

A charged conducting sphere with a radius of **5 cm** has a potential **of 8 kilovolts** relative to infinity. What is the electric field energy density (in $\mathbf{J/m^3}$) at a point near the surface outside the conductor?

Answer: **0.11**

Solution: At the surface of the conductor we know that,

$$V_{surface} = \frac{KQ}{R} \quad E_{surface} = \frac{KQ}{R^2}.$$

Hence, the electric field energy density is

$$u = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0 \frac{V^2}{R^2}$$
$$= \frac{(8.85\times10^{-12}C^2/(Nm^2))(8\times10^3 V)^2}{2(5\times10^{-2}m)^2} = 0.11 J/m^3$$

Problem 5:

The electric field energy density near the surface of an isolated charged conducting sphere with a radius of **5 cm** is $\mathbf{0.1\ J/m^3}$. What is the electric potential (**in kiloVolts**) at the center of the conductor?

Answer: **7.5**

Solution: At the surface of the conductor we know that,

$$V_{surface} = \frac{KQ}{R} \quad E_{surface} = \frac{KQ}{R^2}$$

and the electric field energy density is

$$u = \frac{1}{2}\varepsilon_0 E^2 .$$

Thus,

$$V_{center} = V_{surface} = R\sqrt{\frac{2u}{\varepsilon_0}}$$

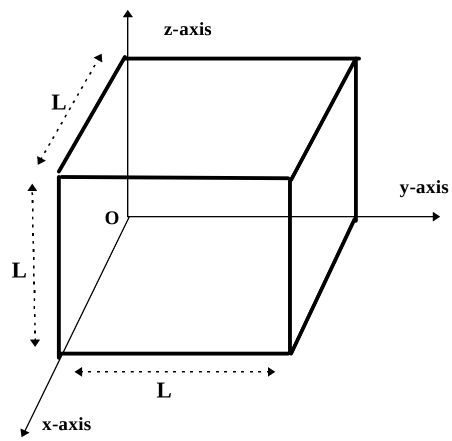
$$= (0.05m)\sqrt{\frac{2(0.1J/m^3)}{8.85\times10^{-12}C^2/(Nm^2)}} = 7.5kV$$

where I have used the fact that the potential is constant throughout the conductor.

Problem 6:

In a certain region of space the electric field is given by, $\vec{E} = Ay\hat{x}$,

where **A=100 V/m^2**. The electric field points in the x direction and has a constant magnitude given by **E(y)=Ay**. What is the total amount of electric potential energy (**in milliJoules**) contained in the electric field within the cube with sides of length **L=20 meters** shown in Figure?



Answer: 47.2

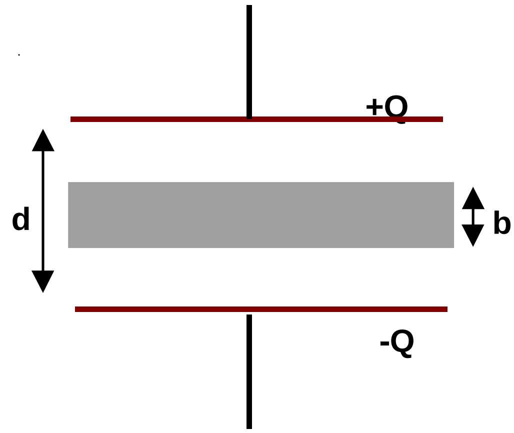
Solution: The energy stored in the electric field lines is given by

$$U = \int u dV = \int_0^L \frac{1}{2}\varepsilon_0 E^2 L^2 dy = \frac{1}{2}\varepsilon_0 A^2 L^2 \int_0^L y^2 dy = \frac{1}{6}\varepsilon_0 A^2 L^5 ,$$

and plugging in the numbers gives

U = (8.85x10^{-12} C^2/(Nm^2))(100 V/m^2)^2(20 m)^5/6 = 47.2 mJ.

Problem 7:



Given an open-air parallel-plate capacitor of plate area **A** and plate separation **d** as shown in the **Figure**. A flat slice of conducting material (thickness **b**; **b<d**) of the same area **A** is now carefully inserted between the plates parallel to them without touching. If the charge **Q** on the capacitor was unchanged during this operation, must the metal slice be forced between the plates or will it be pulled in by electrostatic forces, and what is the new capacitance?

Answer: $\varepsilon_0 A/(d-b)$**, pulled**

Solution: Initially the capacitor has $E = Q/(\varepsilon_0 A)$, $\Delta V = Ed$, $C_i = \varepsilon_0 A/d$, and stored energy

$$U_i = \frac{Q^2}{2C_i} = \frac{Q^2 d}{2\varepsilon_0 A}.$$

After the conducting material is inserted, we have two capacitors in series with

$$C = \frac{\varepsilon_0 A}{\left(\frac{d}{2} - \frac{b}{2}\right)} = \frac{2\varepsilon_0 A}{(d-b)} \quad \text{and} \quad C_{eff} = \frac{C}{2} = \frac{\varepsilon_0 A}{(d-b)}.$$

The stored energy is now

$$U_f = \frac{Q^2}{2C_{eff}} = \frac{Q^2(d-b)}{2\varepsilon_0 A} < U_i.$$

Since the final stored energy is less than the initial stored energy, the conducting material will be pulled in by the electrostatic force with the difference in stored energy doing the work.

Chapter 27 Solutions

Problem 1:

Wire **B** is three times longer than wire **A**, and the two wires have the **same volume**. If both wires are made of the same substance and if the resistance of wire **A** is **9 Ohms**, what is the resistance of wire **B** (**in Ohms**).

Answer: 81

Solution: We know that the resistance of a wire of length **L**, cross sectional area **A**, and resistivity **ρ** is given by

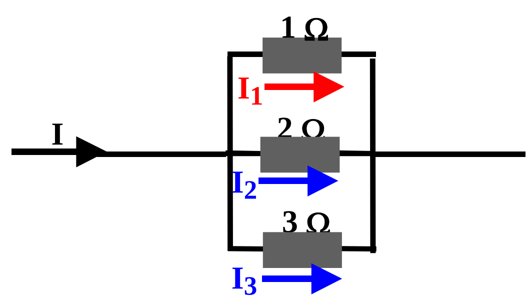
$$R = \frac{\rho L}{A} = \frac{\rho L^2}{V},$$

where **V** is the volumn, **V = LA**. Thus,

$$R_B = \frac{\rho L_B^2}{V_B} = \frac{\rho(3L_A)^2}{V_A} = 9\frac{\rho L_A^2}{V_A} = 9R_A = 9(9\Omega) = 81\Omega.$$

Problem 2:

How much current passes through the **1 Ω** resistor in the circuit shown in the **Figure**?



Answer: 6I/11

Solution: Since the potential drop across each resistor is the same (i.e. parallel) we know that

$$I_1R_1 = I_2R_2 = I_3R_3 \quad \text{and in addition} \quad I_1 + I_2 + I_3 = I.$$

Hence,

$$I_1 + \frac{R_1I_1}{R_2} + \frac{R_1I_1}{R_3} = I_1\left(1 + \frac{R_1}{R_2} + \frac{R_1}{R_3}\right) = I$$

and solving for I_1 gives

$$I_1 = \frac{I}{\left(1 + \frac{R_1}{R_2} + \frac{R_1}{R_3}\right)} = \frac{I}{\left(1 + \frac{1}{2} + \frac{1}{3}\right)} = \frac{6}{11}I$$

Problem 3:

Consider a long straight cylindrical wire with radius **R** carrying the ***non-uniform*** current density, **J(r)=ar²**, where **r** is the distance from the center axis of the wire. How much total current **I** passes through the wire?

Answer: **$\pi aR^4/2$**

Solution: The current **I** is the "flux" associated with the current density as follows:

$$I = \int \vec{J} \cdot d\vec{A} = \int_0^R \left(ar^2\right) 2\pi r dr = 2\pi a \int_0^R r^3 dr = \frac{1}{2}\pi a R^4 .$$

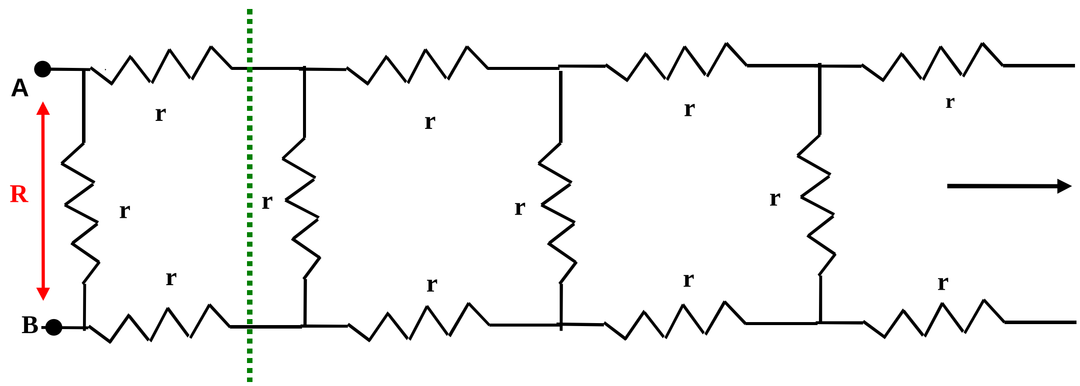
Problem 4:

Consider a long straight cylindrical wire with radius **R = 0.1 m** is carrying the ***non-uniform*** current density which points along the axis of the wire and has a magnitude, **J(r) = a/r**, where **r** is the distance from the center axis of the wire. If **a = 2 A/m**, how much total current **I** passes through the wire (**in A**)?

Answer: **1.26**

Solution: The current **I** is the "flux" associated with the current density as follows:

$$I = \int \vec{J} \cdot d\vec{A} = \int_0^R \left(\frac{a}{r}\right) 2\pi r dr = 2\pi a R = 2\pi(2A/m)(0.1m) = 1.26A .$$

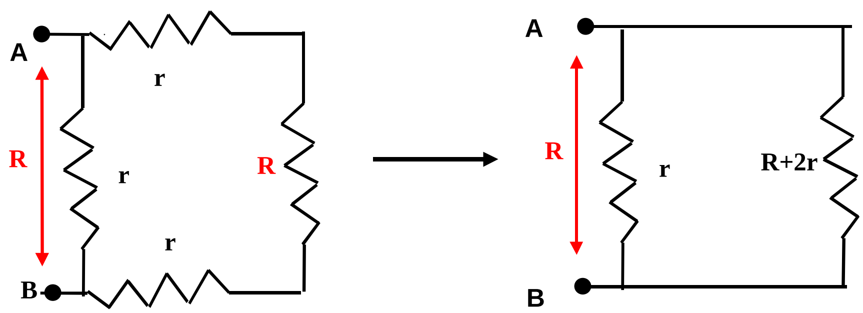


Problem 5:

Consider the infinite chain of resistors shown in the **Figure**. Calculate the effective resistance, **R**, (**in Ohms**) of the network between the terminals **A** and **B** given that each of the resistors has resistance **r = 1 Ohm**.

Answer: 0.7

Solution: We first cut the infinite chain at the point shown in the figure and replace the infinite chain to the right of the cut by its effective resistance **R**. We then combine



the three resistors in series and then combine **r** and **R+2r** in parallel giving,

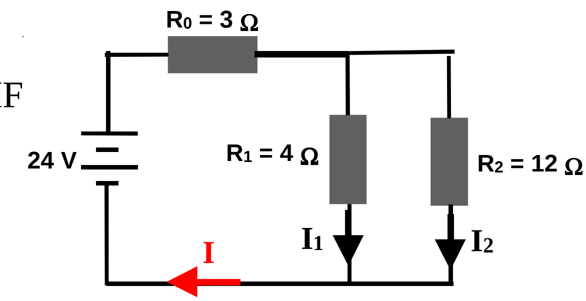
$$\frac{1}{R}=\frac{1}{r}+\frac{1}{R+2r},$$

which implies that

$$R^2+2rR-2r^2=0 \quad \text{and} \quad R=\left(-1+\sqrt{3}\right)r=0.73r.$$

Chapter 28 Solutions

Problem 1:



Consider the circuit consisting of an EMF and three resistors shown in the **Figure**. How much current flows through the **4 Ω** resistor (**in Amps**)?

Answer: 3

Solution: The original circuit is equivalent to a circuit with and EMF and one resistor with resistance $\mathbf{R_{tot}}$ given by

$$R_{tot} = 3\Omega + R_{eff} = 3\Omega + 3\Omega = 6\Omega,$$

where $\mathbf{R_{eff} = 3\Omega}$ from

$$\frac{1}{R_{eff}} = \frac{1}{4\Omega} + \frac{1}{12\Omega} = \frac{1}{3\Omega}.$$

Thus, the current $\mathbf{I = 24V/6\Omega = 4A}$. Now, we know the $\mathbf{I = I_1 + I_2}$ and that $\mathbf{\Delta V_1 = I_1 R_1 = \Delta V_2 = I_2 R_2}$ and hence

$$I_1 = \frac{R_2}{R_1 + R_2} I = \frac{12}{16}(4A) = 3A.$$

Problem 2:

A constant power **P** is supplied to a transmission line of resistance **R** by a power station. If the power dissipated in the transmission line is **200 Watts** when the power station delivers an **EMF** of **10 Volts**, how much power is dissipated in the transmission line (**in W**) when the power station delivers an **EMF** of **20 Volts** *(assuming the same power is supplied in both cases)*?

Answer: 50

Solution: The powers supplied by the station is given by

$$P_{station} = I_1 V_1 = I_2 V_2,$$

is constant so that

$$\frac{I_2}{I_1} = \frac{V_1}{V_2},$$

where V_1 **= 10 V** and V_2 **= 20 V**. Now the power disapated in the line is $P = I^2R$ so that

$$P_2 = \left(\frac{I_2}{I_1}\right)^2 P_1 = \left(\frac{V_1}{V_2}\right)^2 P_1 = \frac{P_1}{4} = 50W \;.$$

The power station loses less energy in the line if it transmits power at a higher voltage!

Problem 3:

A capacitor **C** with an initial charge **Q** discharges through a resistor **R**. How many time constants $\tau = RC$ must elapse in order for the capacitor to lose **2/3** of its charge?

Answer: 1.1

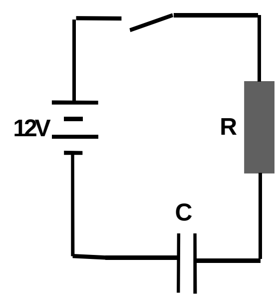
Solution: For an **RC** circuit with the charge **Q(t)** given by

$$Q(t) = Q_0 e^{-t/\tau} \quad \text{and}$$

$$t = -\tau \log(Q(t)/Q_0) = -\tau \log(1/3) = 1.1\tau \,,$$

where $\tau = RC$ and where I used $Q(t)/Q_0 = 1/3$.

Problem 4:

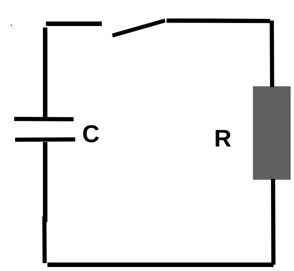


The capacitor in the Figure is initially uncharged. After the switch is closed (**EMF = 12 V**), what is the magnitude of the potential difference across the capacitor (**in V**) when the charge on the capacitor reaches one-third of its maximum value?

Answer: 4

Solution: The potential difference across the is $\Delta V_C = Q/C = \varepsilon C/(3C) = \varepsilon/3 = 4$ V, where I used $Q_{max} = \varepsilon C$.

Problem 5:



A charged capacitor **C** has a resistance **R** connected across its terminals to form the **RC circuit** shown in the Figure. If it takes **2 seconds** for the capacitor to loose one-half of its stored energy, how long does it take (**in s**) for it to loose **90%** of its initial charge?

Answer: 13.3

Solution: The stored energy in the **RC** circuit is given by

$$U(t) = \frac{Q(t)^2}{2C} = U_0 e^{-2t/\tau},$$

and solving for **τ** gives

$$\tau = RC = \frac{-2t_1}{\ln(U(t)/U_0)} = \frac{-2t_1}{\ln(0.5)},$$

where $\mathbf{t_1 = 2\ s}$. The charge as a function of time is

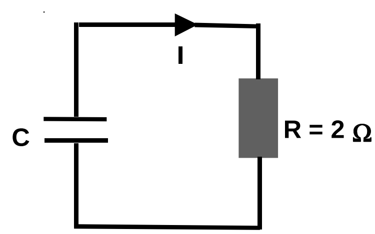
$$Q(t) = Q_0 e^{-t/\tau} \quad \text{and} \quad t = -\tau \ln(Q(t)/Q_0).$$

Thus,

$$t = \frac{2t_1 \ln(0.1)}{\ln(0.5)} = \frac{2(2s)\ln(0.1)}{\ln(0.5)} = 13.3s.$$

Problem 6:

A capacitor, **C**, is (*fully*) charged by connecting it to a **10 Volt** EMF and then is allowed to discharge through a **2 Ω** resistor as shown in the **Figure**? What is the current, **I**, in the circuit (**in A**) when the stored energy in the capacitor has dropped to **50%** of its initial value?



Answer: 3.5

Solution: The charge, **Q(t)**, and stored energy, **U(t)**, as a function of time is given by

$$Q(t) = Q_0 e^{-t/RC} = Q_0 e^{-t/\tau}$$

$$U(t) = \frac{Q^2}{2C} = U_0 e^{-2t/RC} = U_0 e^{-2t/\tau},$$

where $\mathbf{U_0 = Q_0^2/(2C)}$ and $\mathbf{Q_0 = CV_0}$. Here $\mathbf{V_0}$ is the potential across the capacitor at **t = 0** (equal to the EMF that fully charged the capacitor). If $\mathbf{t_a}$ is the time when the stored energy reaches **50%** of its initial value, then

$$e^{-2t_a/\tau} = \frac{1}{2} \quad \text{and taking the square-root gives} \quad e^{-t_a/\tau} = \frac{1}{\sqrt{2}}.$$

The current at time $\mathbf{t_a}$ is given by

$$I(t_a) = I_0 e^{-t_a/\tau} = \left(\frac{V_0}{R}\right)\frac{1}{\sqrt{2}} = \frac{10V}{2\Omega}\frac{1}{\sqrt{2}} = \frac{5}{\sqrt{2}}A = 3.5A,$$

where I used $\mathbf{I_0 = V_0/R}$.

Problem 7:

A "leaky" capacitor's faulty insulation allows charge to pass slowly from one plate to the other. Suppose that a charged leaky **2.0 microF** capacitor's potential difference drops to one-fourth its initial value in **2.0 sec**. What is the equivalent resistance between the capacitor plates, in megaOhms? (*Hint: the leaky capacitor forms an RC circuit.*)

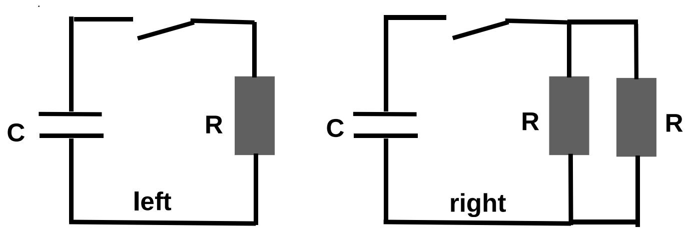
Answer: 0.72

Solution: The leaky capacitor forms an **RC** circuit with the charge **Q(t)** given by

$$Q(t) = Q_0 e^{-t/\tau} \quad \text{and} \quad \tau = RC = \frac{-t}{\ln(Q(t)/Q_0)}.$$

Solving for the resistance **R** gives

$$R = \frac{-t}{C\ln(Q(t)/Q_0)} = \frac{-(2s)}{(2\times10^{-6}F)\ln(0.25)} = 0.721M\Omega.$$



Problem 8:

Consider the discharging of the two capacitors shown in the **Figure**. Both capacitors have equal capacitance **C** and both have an initial charge of **8 microC**. When the switches are closed the capacitor an the left will discharge through a single resistor with resistance **R**, while the capacitor on the right discharges through two equal resistors with resistance **R** connected in parallel. If the switches are closed simultaneously, how much charge is on the right capacitor (**in microC**) when the left capacitor's charge is **4 microC**?

Answer: **2**

Solution: The charge as a function of time on the two capacitors is given by

$$Q_{left}(t) = Q_0 e^{-t/RC}$$
$$Q_{right}(t) = Q_0 e^{-2t/RC} \ ,$$

where I used the fact the the effective resistance on the right side is $\mathbf{R_{eff} = R/2}$. Hence,

$$\left(\frac{Q(t)}{Q_0}\right)_{right} = \left(\frac{Q(t)}{Q_0}\right)^2_{left} .$$

Thus, when the left side $\mathbf{Q(t)/Q_0 = 4/8 = 1/2}$ the right side has $\mathbf{Q(t)/Q_0 = 1/4}$ so that $\mathbf{Q_{right} = 2\ \mu C}$.

Chapter 29 Solutions

Problem 1:

A charged particle, **Q = 0.5 C**, enters a region with a uniform magnetic field $\vec{B} = 2\hat{x} + 3\hat{y} + 4\hat{z}$ (**in Tesla**). If its velocity is given by $\vec{v} = 2\hat{x} + 3\hat{y} + 2\hat{z}$ (**in m/s**), what is the magnitude of the magnetic force on the particle (**in N**)?

Answer: 3.6

Solution: We know that the magnetic force on a particle is

$$\vec{F} = Q\vec{v}\times\vec{B} = Q\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 2 & 3 & 2 \\ 2 & 3 & 4 \end{vmatrix} = Q(6\hat{x} - 4\hat{y}).$$

The **magnitude of the force** is

$$F = Q\sqrt{36+16} = (0.5)\sqrt{52}\,N = 3.6\,N.$$

Problem 2:

A charged particle, **Q=0.5 C**, and mass **M = 200 grams** enters a region with a uniform magnetic field $\vec{B} = 2\hat{x} - 1\hat{y}$ (**in Tesla**). What is the magnitude of the magnetic force on the particle (**in N**), if its velocity is $\vec{v} = 4\hat{x} - 2\hat{y}$ (**in m/s**)?

Answer: zero

Solution: We know that the magnetic force on a particle is

$$\vec{F} = Q\vec{v}\times\vec{B} = Q\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 4 & -2 & 0 \\ 2 & -1 & 0 \end{vmatrix} = 0.$$

Note that **v** is in the same direction as **B** and hence the magnetic force is zero.

Problem 3:

One end of a straight wire segment is located at (x,y,z) = **(0,0,0)** and the other end is at **(1m,2m,3m)**. A current of **2 Amps** flows through the segment. The segment sits in a uniform magnetic field $\vec{B} = 2\hat{x} - 1\hat{y}$ (**in**

Tesla). What is the magnitude of the magnetic force (**in N**) on the wire segment?

Answer: 16.7

Solution: We know that the magnetic force on a wire segment is

$$\vec{F} = I\vec{L} \times \vec{B} = I\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 1 & 2 & 3 \\ 2 & -1 & 0 \end{vmatrix} = I(3\hat{x} + 6\hat{y} - 5\hat{z})Tm\ ,$$

with the **magnitude of the force** given by

$$\left|\vec{F}\right| = I\sqrt{9+36+25}\,Tm = (2A)8.367\,Tm = 16.73\,N\ .$$

Problem 4:

An electric power transmission line located an average distance of **20 m** above the earth's surface carries a current of **800 Amps** from east to west, in a region where the earth's magnetic field is **0.8 gauss** due north at **60^o** below the horizontal. What is the magnitude of the force per meter on the line (**in milliN/m**)?

Answer: 64

Solution: We know that the force on a straight wire is given by

$$\vec{F} = I\vec{L} \times \vec{B}\ .$$

In this case **L** and **B** are perpendicular and hence,

$$\frac{F}{L} = IB = (800\,A)(0.8\times10^{-4}\,T) = 64\times10^{-3}\,N/m\ .$$

Problem 5:

A charged particle, **Q=0.1 C**, traveling in the x-direction with velocity $\vec{v} = v_0\hat{x}$ enters a region of space that has an electric field in the y-direction given by $\vec{E} = E_0\hat{y}$ with **E_0 = 10 Volts** and a magnetic field in the z-direction given by $\vec{B} = B_0\hat{z}$ with **B_0 = 0.2 Tesla**. If the particle experiences no ***net*** force and continues with the same speed and direction, what is its speed **v_0 (in m/s)**?

Answer: 50

Solution: The electric force is given by

$$\vec{F}_E = QE_0\hat{y}$$

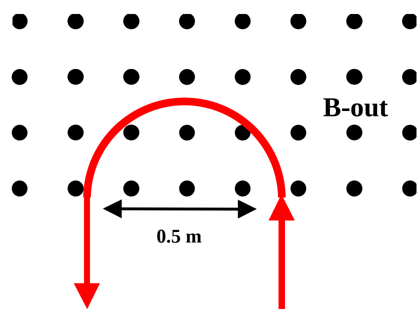
and the magnetic force is

$$\vec{F}_E = Q\vec{v}\times\vec{B} = -Qv_0B_0\hat{y}$$

and setting $\vec{F}_E + \vec{F}_B = 0$ implies that

$$v_0 = \frac{E_0}{B_0} = \frac{10V}{0.2T} = 50\,m/s\,.$$

Problem 6:



A charged particle traveling in the y-direction with a momentum of **0.01 kg m/s** enters a region of space that has a uniform **1 Tesla** magnetic field in the z-direction as shown the **Figure**. If the particle enters the magnetic field at the point A and then exits the magnetic field at the point B located a distance of **0.5 m** to the left of the point A, what is the charge of the particle (**in milliC**)?

Answer: -40

Solution: The radius of the circular path followed by the charged particle calculated by setting the force equal to the mass times the centripetal acceleration as follows,

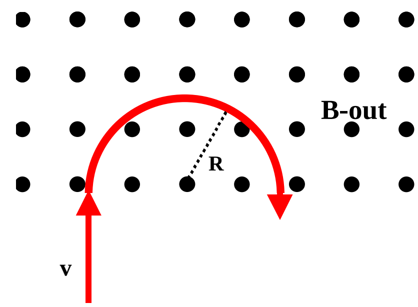
$$|Q|vB = \frac{mv^2}{R}\,,$$

and solving for the magnitude of the charge gives

$$|Q| = \frac{mv}{RB} = \frac{2p}{dB} = \frac{2(0.01kgm/s)}{(0.5m)(1T)} = 40\,mC\,,$$

where I used **p = mv** and **d = 2R**. We know that the sign of the charge must be negative since the particle moves to the left.

Problem 7:



A charged particle traveling in the y-direction enters a region of space that has a uniform **2 Tesla** magnetic field in the z-direction as shown in the **Figure**. If the particle has a charge of **0.1 C** and a mass of **0.2 kg** how long (**in seconds**) does it take for the particle to reverse direction and exit the region?

Answer: 3.1

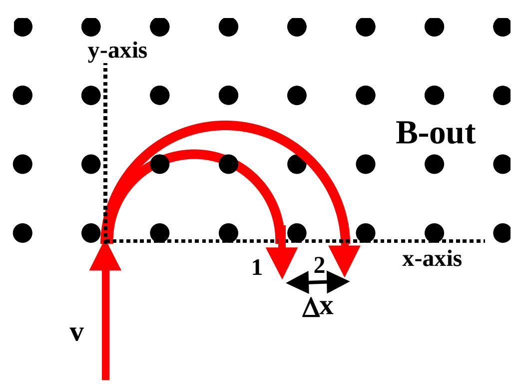
Solution: The radius of the **circular path** followed by the charged particle calculated by setting the force equal to the mass times the centripetal acceleration as follows,

$$QvB = \frac{mv^2}{R}$$, and solving for the radius gives $$R = \frac{mv}{QB}.$$

The time is given by the distance traveled divided by the speed as follows:

$$t = \frac{\pi R}{v} = \frac{\pi m}{QB} = \frac{\pi(0.2kg)}{(0.1C)(2T)} = 3.14s.$$

Problem 8:



Particle **#1** and particle **#2** travel along the y-axis and enter a region of space that has a uniform **2 Tesla** magnetic field in the z-direction as shown in the **Figure**. Both particles have the same charge, **Q=0.1 C**, and both have the same speed, **v = 10 m/s**. If the particles are a distance $\boldsymbol{\Delta x = x_2 - x_1 =}$ **0.3** meters apart when they exit the region, what is the difference in their mass, $\boldsymbol{\Delta M = M_2 - M_1}$ (**in grams**)?

Answer: 3

Solution: The radius of the **circular path** followed by a charged particle is calculated by setting the force equal to the mass times the centripetal acceleration as follows,

$$QvB = \frac{Mv^2}{R}$$, and solving for the radius gives $$R = \frac{Mv}{QB}.$$

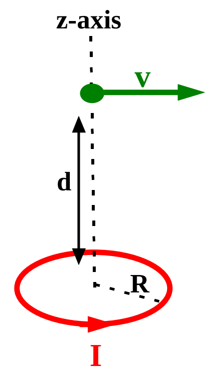
Thus,

$$\Delta M = M_2 - M_1 = \frac{QB}{v}(R_2 - R_1) = \frac{QB}{2v}\Delta x$$

$$= \frac{(0.1C)(2T)}{2(10m/s)}(0.3m) = 3g$$

where I used $\Delta x = x_2 - x_1 = 2R_2 - 2R_1 = 2\Delta R$.

Problem 9:



A circular loop of wire of radius **R = 1 m** is carrying a current of **2 Amps** as shown in the Figure, A particle with charge **Q = 3x10^{-3} C** is on the axis of the loop (z-axis) a distance of **d = 0.5 meters** away from the loop and is moving with a speed of **2x10^{6} m/s** along the x-axis (i.e. perpendicular to the axis of the loop). What is the magnitude of the magnetic force (**in milliN**) on the particle due to the loop?

Answer: 5.4

Solution: The magnetic field on the z-axis of the loop a distance **d = 0.5** meters from the loop is

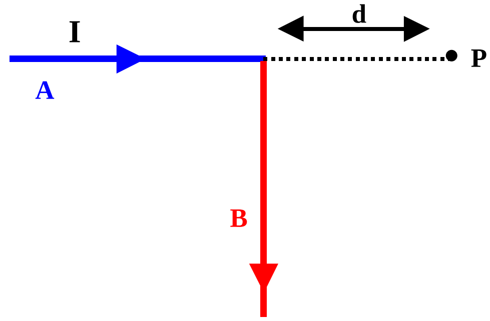
$$B_z = \frac{2kI\pi R^2}{(z^2+R^2)^{3/2}} = \frac{(2\times10^{-7}Tm/A)(2A)\pi(1m)^2}{\left((0.5m)^2+(1m)^2\right)^{3/2}} = 8.99\times10^{-7}T\ .$$

The magnitude of the magnetic force on the particle is

$$\left|\vec{F}\right| = QvB = (3\times10^{-3}C)(2\times10^{6}m/s)(8.99\times10^{-7}T) \approx 5.4mN\ .$$

Chapter 30 Solutions

Problem 1:

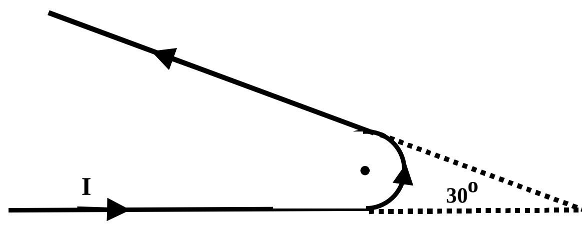


The bent wire carries current **I**, as shown the **Figure**. What is the magnetic field at point **P**, at distance **d** directly opposite the **90°** bend (k = $\mu_0/4\pi$)?

Answer: kI/d, out of page

Solution: Wire **A** does not contribute at **P** since $d\vec{l} \times \hat{r} = 0$ and wire **B** (semi-infinite wire) contributes **kI/d** (one half of 2kI/d).

Problem 2:



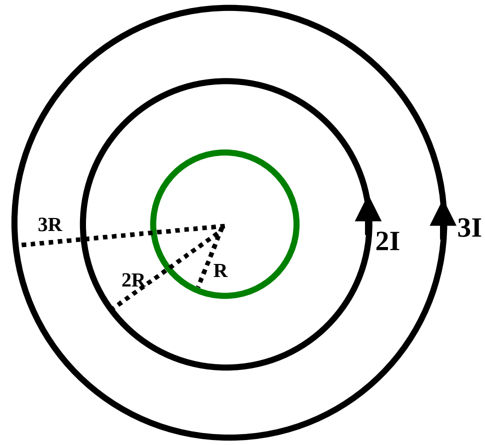
Two semi-infinite straight pieces of wire are connected by a circular loop segment of radius **R** as shown in the **Figure**. If extrapolated to their intersection point straight wires would form a **30^{o}** angle. If the wires carry a current **I**, what is the magnitude of the magnetic field at the center of the circular arc (k = $\mu_0/4\pi$)?

Answer: 4.62kI/R

Solution: The magnetic field at the center of the circular loop segment is the superposition of three terms as follows:

$$B_z = \frac{kI}{R} + \frac{kI}{R} + \left(\frac{150^\circ \pi}{180^\circ}\right)\frac{kI}{R} = (2 + 2.62)\frac{kI}{R} = 4.62\frac{kI}{R}.$$

Problem 3:



Three concentric current loops have radii **R**, **2R**, and **3R**, as shown in the **Figure**. The loop with radius **2R** carries a current of **2I** and the loop with radius **3R** carries a current of **3I** in the same direction (**counter-clockwise**). If the ***net*** magnetic field at the center of the three loops is zero, what is the magnitude and direction of the current in the loop with radius **R**?

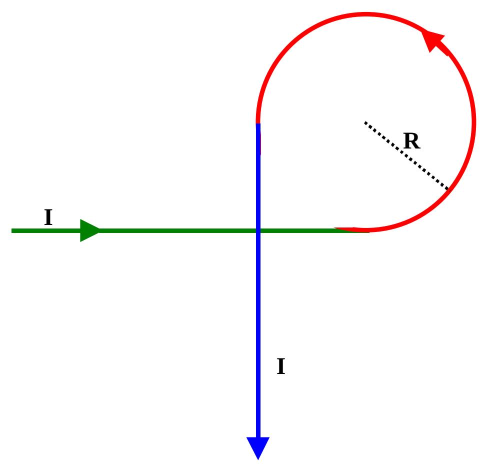
Answer: 2I, clockwise

Solution: If we define the z-axis to be out of the paper then the net magnetic field at the center of the three circles is given by

$$B_z = -\frac{2\pi k I_1}{R} + \frac{2\pi k(2I)}{2R} + \frac{2\pi k(3I)}{3R} = 0,$$

where $\mathbf{I_1}$ is the current (**clockwise**) in the smallest loop. Solving for $\mathbf{I_1}$ gives $\mathbf{I_1 = 2I}$.

Problem 4:



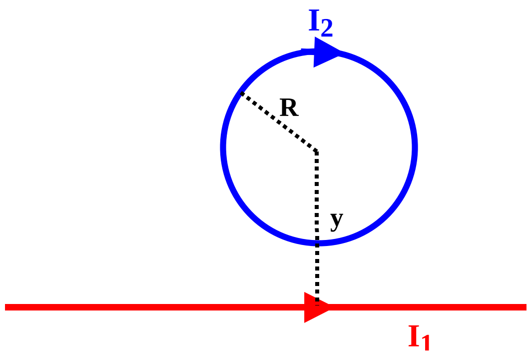
Two semi-infinite straight pieces of wire are connected by a circular loop segment of radius **R** as shown in the **Figure**. If the two straight pieces of wire are perpendicular to each other and if the circuit carries a current **I**, what is the magnitude of the magnetic field at the center of the circular arc ($k = \mu_0/4\pi$)?

Answer: 6.71kI/R

Solution: If we define the z-axis to be out of the paper then the net magnetic field at the center of the circular arc is given by

$$B_z = \frac{kI}{R} + \frac{kI}{R} + \frac{3\pi}{2}\frac{kI}{R} = \left(2 + \frac{3\pi}{2}\right)\frac{kI}{R} = 6.71\frac{kI}{R}.$$

Problem 5:



An infinitely long straight wire carries a current $\mathbf{I_1}$, and a wire loop of radius **R** carries a current $\mathbf{I_2}$ as shown in the **Figure**. If $\mathbf{I_1 = 4I_2}$, and if the ***net*** magnetic field at the center of the circular loop is zero, how far is the center of the loop from the straight wire?

Answer: 1.27R

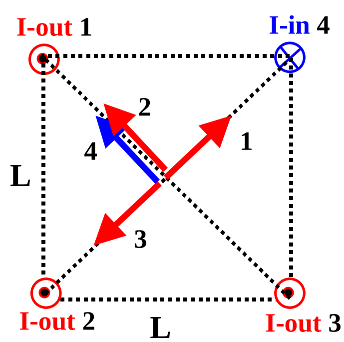
Solution: Setting magnitudes of the magnetic field from the loop and wire equal to each other yields

$$\frac{2kI_1}{y} = \frac{2\pi kI_2}{R}.$$

Solving for **y** gives

$$y = \frac{I_1}{\pi I_2}R = \frac{4}{\pi}R = 1.27R.$$

Problem 6:



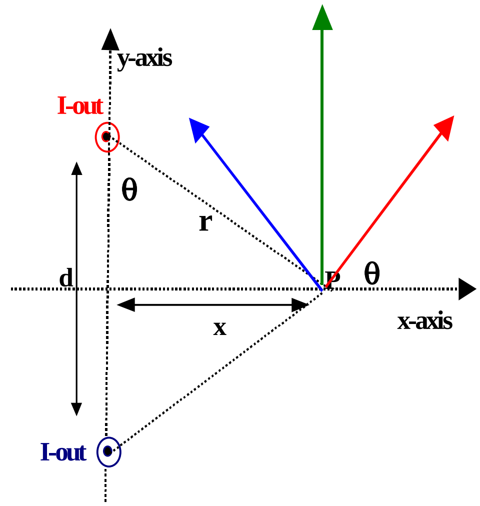
Four infinitely long parallel wires each carrying a current **I** form the corners of square with sides of length **L** as shown in the **Figure**. If three of the wire carry the current **I** in the same direction (**out of the page**) and one of the wires carries the current **I** in the opposite direction (**into the page**), what is the ***net*** magnetic field at the center of the square ($k = \mu_0/4\pi$)?

Answer: 5.7kI/L

Solution: From the Figure that the magnetic fields from wires 1 and 3 cancel and the magnetic fields from wires 2 and 4 add. Thus, the **net** magnetic field is given by

$$B = 2\frac{2kI}{L/\sqrt{2}} = 4\sqrt{2}\frac{kI}{L} = 5.67\frac{kI}{L}.$$

Problem 7:



Two infinitely long parallel wires are a distance **d** apart and carry equal parallel currents **I** in the same direction as shown the **Figure**. If the wires are located on the y-axis at **y = d/2** and **y = -d/2**, what is the distance **x** to the point **P** on the positive x-axis where the magnitude of the magnetic field is maximum?

Answer: d/2

Solution: The magnetic field at the point **P** is the **vector superposition** of the magnetic field from each of the two wires as follows:

$$\vec{B}_1 = \frac{2kI}{r}\left(\cos\theta\hat{x} + \sin\theta\hat{y}\right)$$

$$\vec{B}_2 = \frac{2kI}{r}\left(-\cos\theta\hat{x} + \sin\theta\hat{y}\right)$$

where $\mathbf{r^2 = x^2 + (d/2)^2}$. Hence, the net magnetic field is in the **positive y direction** with magnitude

$$B_y = \frac{4kI}{r}\sin\theta = \frac{4kIx}{r^2} = \frac{4kIx}{(x^2 + (d/2)^2)},$$

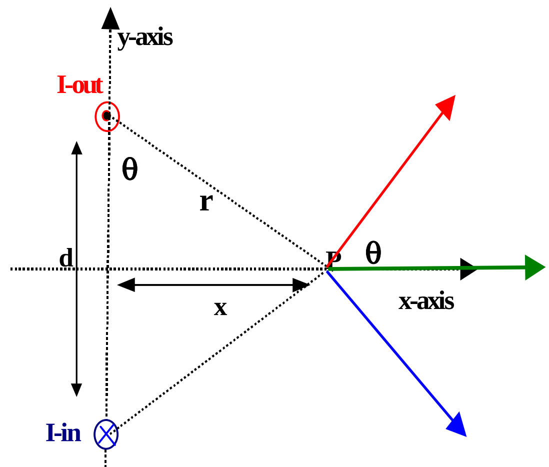
where I used $\mathbf{\sin\theta = x/r}$. To find the point where **B** is maximum we take the derivative as follows,

$$\frac{dB_y}{dx} = 4kI\left(\frac{-2x^2}{(x^2 + (d/2)^2)^2} + \frac{1}{(x^2 + (d/2)^2)}\right) = 0,$$

which implies that

$$-2x^2 + x^2 + (d/2)^2 = 0 \quad \text{and} \quad x = d/2.$$

Problem 8:



Two infinitely long parallel wires are a distance **d** apart and carry equal antiparallel currents **I** as shown in the Figure. If the wires are located on the y-axis at **y=+d/2** and **y=-d/2**, what is the magnitude of the magnetic field at a point **x** on the x-axis (assume that **x** >> **d** and let $k = \mu_0/4\pi$)?

Answer: $2kId/x^2$

Solution: The magnetic field at the point **P** is the **vector superposition** of the magnetic field from each of the two wires as follows:

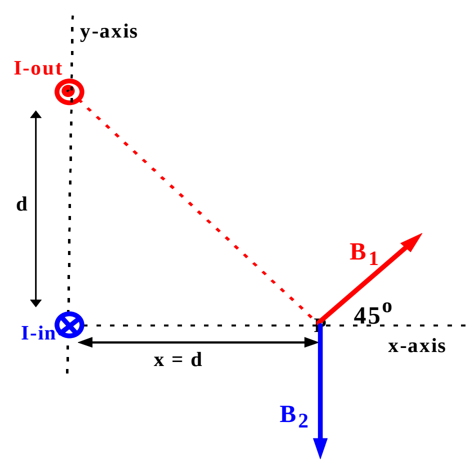
$$\vec{B}_1 = \frac{2kI}{r}\left(\cos\theta\hat{x} + \sin\theta\hat{y}\right)$$
$$\vec{B}_2 = \frac{2kI}{r}\left(\cos\theta\hat{x} - \sin\theta\hat{y}\right)$$

where $r^2 = x^2 + (d/2)^2$. Hence, the net magnetic field is in the **positive x direction** with magnitude

$$B_x = \frac{4kI}{r}\cos\theta = \frac{2kId}{r^2} = \frac{2kId}{(x^2+(d/2)^2)} \rightarrow \frac{2kId}{x^2},$$

where I used $\cos\theta = d/(2r)$.

Problem 9:



Two infinitely long parallel wires are a distance **d** apart and carry equal antiparallel currents **I** as shown in the Figure. If the wires are located on the y-axis at **y = 0** and **y = d**, what is the magnitude of the magnetic field at the point **x = d** on the x-axis?

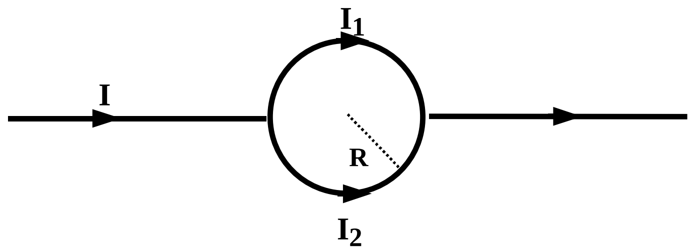
Answer: $\sqrt{2}kI/d$

Solution: The magnetic field at P is the superposition of the fields from each wire as follows:

$$\vec{B}_1 = \frac{2kI}{\sqrt{2}d}\left(\frac{1}{\sqrt{2}}\hat{x} + \frac{1}{\sqrt{2}}\hat{y}\right)$$
$$\vec{B}_2 = -\frac{2kI}{d}\hat{y}$$
$$\vec{B}_{tot} = \vec{B}_1 + \vec{B}_2 = \frac{kI}{d}(\hat{x} - \hat{y})$$

with $\left|\vec{B}_{tot}\right| = \sqrt{2}kI/d$.

Problem 10:



A single infinitely long straight piece of wire carrying a current **I** is split and bent so that it includes two half circular loops of radius **R**, as shown in the **Figure**. If current $\mathbf{I_1}$ goes through the top loop and current $\mathbf{I_2}$ through the lower loop (with $\mathbf{I = I_1+I_2}$) and if $\mathbf{I_2 = 2I_1}$ what is the magnitude of the magnetic field at the center of the circle ($k = \mu_0/4\pi$)?

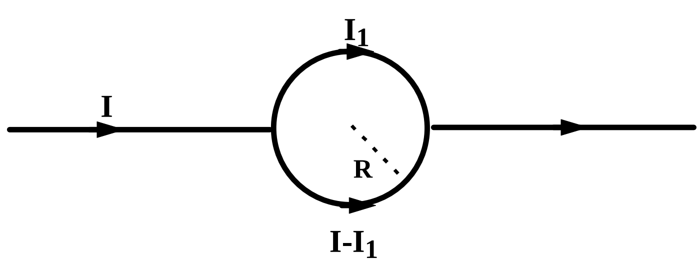
Answer: $\pi kI/(3R)$

Solution: The magnetic field at the center of the loop is

$$B_z = \frac{\pi kI_2}{R} - \frac{\pi kI_1}{R} = \frac{\pi k(I_2 - I_1)}{R} = \frac{\pi kI}{3R},$$

where I used $\mathbf{I_1 = I/3}$ and $\mathbf{I_2 = 2I/3}$.

Problem 11:



A single infinitely long straight piece of wire carrying a current **I** is split and bent so that it includes two half circular loops of radius **R**, as shown in the Figure. If current $\mathbf{I_1}$ goes through the top loop (and $\mathbf{I-I_1}$ through the lower loop) and if the magnetic field at the center of the loops is $\vec{B} = \pi kI/(2R)\hat{z}$ where the z-axis is out of the paper, what is the current $\mathbf{I_1}$?

Answer: I/4

Solution: The magnetic field at the center of the loops is

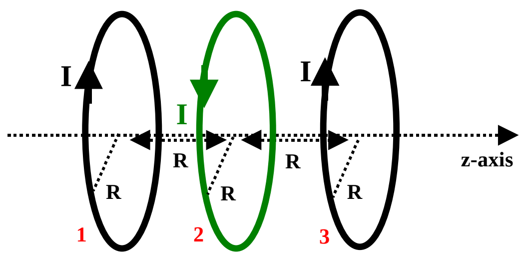
$$B_z = \frac{\pi k(I - I_1)}{R} - \frac{\pi k I_1}{R} = \frac{\pi k(I - 2I_1)}{R} = \frac{\pi k I}{2R}.$$

Hence,

$$I - 2I_1 = \frac{I}{2} \qquad I_1 = \frac{I}{4}.$$

Problem 12:

Three coaxial current loops of radius **R** each carrry a current **I** as shown in the **Figure**. The center loop carries the current in the opposite direction from the outer two loops. If the centers of the current loops are a distance **R** apart what is the magnitude of the magnetic field at the center of the middle current loop ($k = \mu_0/4\pi$)?



Answer: 1.84kI/R

Solution: The magnetic field at the center of the middle loop is the superposition of the magnetic fields from each of the three loops as follows:

$$\vec{B}_1 = -\frac{2\pi k I R^2}{(R^2 + R^2)^{3/2}}\hat{z} = -\frac{\pi k I}{R\sqrt{2}}\hat{z}$$

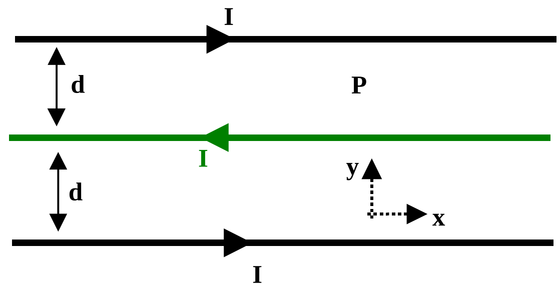
$$\vec{B}_2 = \frac{2\pi k I}{R}\hat{z}$$

$$\vec{B}_3 = -\frac{2\pi k I R^2}{(R^2 + R^2)^{3/2}}\hat{z} = -\frac{\pi k I}{R\sqrt{2}}\hat{z}$$

The **net magnetic field** is given by

$$B_z = \frac{\pi k I}{R}\left(2 - \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right) = 1.84\frac{kI}{R}.$$

Problem 13:



Three infinitely long parallel straight wires lie in the xy-plane as shown in the **Figure**. The middle wire carres a current **I** to the left while the top and bottom wires each carry a current **I** to the right. If the wires are a distance **d** apart, what is the magnitude of the magnetic field at the midpoint **P** between the top and middle wire ($k = \mu_0/4\pi$)?

Answer: 6.67kI/d

Solution: The magnetic field at midpoint **P** between the top and middle wire is the superposition of the magnetic fields from each of the three wires as follows:

$$\vec{B}_{top} = -\frac{2kI}{d/2}\hat{z} = -\frac{4kI}{d}\hat{z}$$
$$\vec{B}_{midddle} = -\frac{2kI}{d/2}\hat{z} = -\frac{4kI}{d}\hat{z}$$
$$\vec{B}_{bottom} = \frac{2kI}{3d/2}\hat{z} = \frac{4kI}{3d}\hat{z}$$

The **net** magnetic field is given by

$$B_z = -\frac{2kI}{d}\left(2+2-\frac{2}{3}\right) = -6.67\frac{kI}{d}.$$

Problem 14:

In the previous problem, what is the y-component of the force per unit length exerted on the top wire due to the other two wires ($k = \mu_0/4\pi$)?

Answer: kI^2/d

Solution: The magnetic field at the top wire due to the other two wires is given by

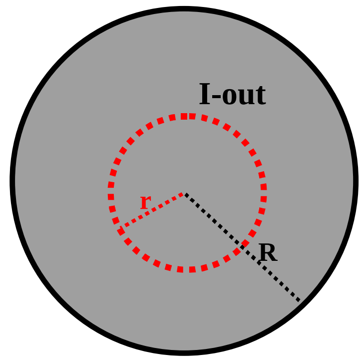
$$\vec{B} = -\frac{2kI}{d}\left(1-\frac{1}{2}\right)\hat{z} = -\frac{kI}{d}\hat{z}$$

and the force on a length **L** of the top wire due to this magnetic field is

$$\vec{F} = I\vec{L} \times \vec{B} = \frac{kI^2L}{d}\hat{y}.$$

The **y-component of the force per unit length** is thus, $\mathbf{F_y/L = kI^2/d}$.

Problem 15:



The **Figure** shows the cross section of a solid cylindrical conductor with radius **R**. The conductor is carrying a uniformly distributed current **I** (out of page). Find the magnitude of the magnetic field, **B**, inside the conductor a distance **r = R/4** from the center ($k = \mu_0/4\pi$).

Answer: kI/(2R)

Solution: If we draw a loop at radius **r** **(r < R)**, then **Ampere's Law** tells us that

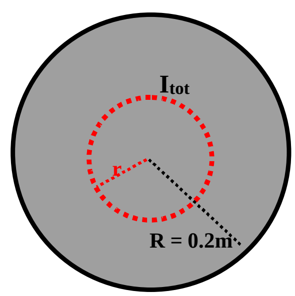
$$\oint_{Loop} \vec{B}\cdot d\vec{l} = 2\pi r B(r) = \mu_0 I_{enclosed}$$, with $I_{enclosed} = \pi r^2 J$ and

$I_{tot} = \pi R^2 J$, where **J** is the (**uniform**) current density. Hence,

$$B(r) = \frac{2kI_{enclosed}}{r} = \frac{2krI_{tot}}{R^2}.$$

Plugging in **r = R/4** gives **B = kI/(2R)**.

Problem 16:



The **Figure** shows the cross section of a solid cylindrical conductor with radius **0.2 m**. The conductor is carrying a uniformly distributed current **I** (out of page). If the magnitude of the magnetic field inside the conductor a distance **0.1 m** from the center is **0.1 microTesla**, what is the total currrent, **I**, carried by the cylindrical conductor (**in Amps**)?

Answer: 0.2

Solution: If we draw a loop at radius **r (r < R)**, then **Ampere's Law** tells us that

$$\oint_{Loop} \vec{B}\cdot d\vec{l} = 2\pi r B(r) = \mu_0 I_{enclosed}$$, with $I_{enclosed} = \pi r^2 J$ and

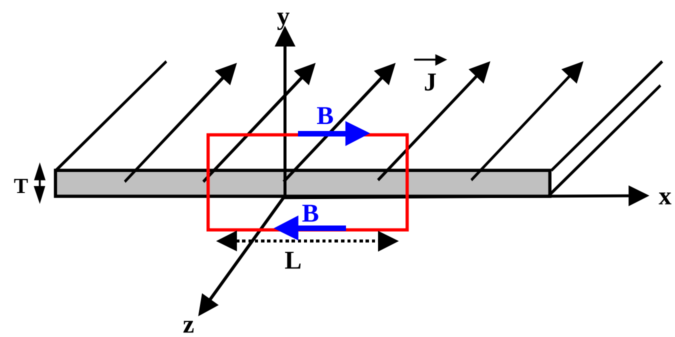
$I_{tot} = \pi R^2 J$, where **J** is the (**uniform**) current density. Hence,

$$B(r) = \frac{2krI_{tot}}{R^2} \quad \text{and} \quad B(R/2) = \frac{kI_{tot}}{R}.$$

Solving for $\mathbf{I_{tot}}$ gives

$$I_{tot} = \frac{RB(R/2)}{k} = \frac{(0.2m)(0.1\times10^{-6}T)}{1\times10^{-7}Tm/A} = 0.2A.$$

Problem 17:



An infinite conducting sheet of thickness **T** lies in the xz-plane and carries a uniformly distributed current density $\vec{J}$ in the **–z** direction as shown in the **Figure**. What is the magnitude and direction of the magnetic field at distance **d** above the sheet (in the **+y** direction, k = $\mu_0/4\pi$)?

Answer: $+0.5\mu_0 JT\hat{x}$

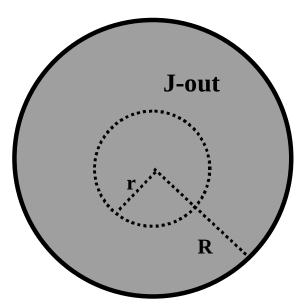
Solution: If we draw an "Amperian" loop (square with sides of length L), then **Ampere's Law** tells us that

$$\oint_{Loop} \vec{B}\cdot d\vec{l} = 2LB = \mu_0 I_{enclosed} = \mu_0 JTL.$$

Solving for **B** gives

$$\vec{B} = 0.5\mu_0 JT\hat{x}.$$

Problem 18:



A long straight cylindrical wire with radius R (see the **Figure**) is carrying a *non-uniform* current density which points along the axis of the wire and has a magnitude, **J(r) = a/r**, where **a** is a constant and **r** is the distance from the center axis of the wire. What is the magnitude of the magnetic field inside the wire a distance **r** from the center (0 < r < R)?

Answer: **$\mu_0 a$**

Solution: If we draw a loop at radius **r (r < R)**, then **Ampere's Law** tells us that

$$\oint_{Loop} \vec{B}\cdot d\vec{l} = 2\pi r B(r) = \mu_0 I_{enclosed} ,$$

with

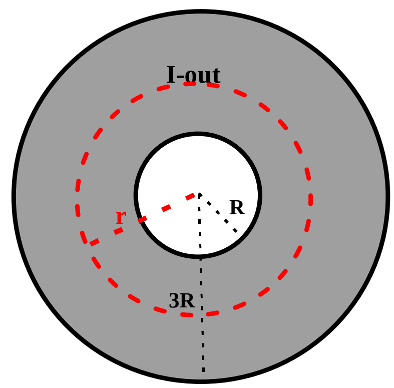
$$I_{enclosed} = \int \vec{J}\cdot d\vec{A} = \int_0^r \frac{a}{r} 2\pi r dr = 2\pi a r .$$

Hence,

$$B(r) = \frac{\mu_0 I_{enclosed}}{2\pi r} = \mu_0 a .$$

Problem 19:

The Figure shows the cross section of a solid cylindrical conductor with radius **3R** with a cylindrical hole of radius **R**. The conductor is carrying a uniformly distributed current **I** (out of page). Find the magnitude of the magnetic field, **B**, at a distance **r = 2R** from the center.



Answer: **3kI/(8R)**

Solution: If we draw a loop at radius **r** **(R < r < 3R)**, then **Ampere's Law** tells us that

$$\oint_{Loop} \vec{B}\cdot d\vec{l} = 2\pi r B(r) = \mu_0 I_{enclosed}$$, with $I_{enclosed} = (\pi r^2 - \pi R^2)J$

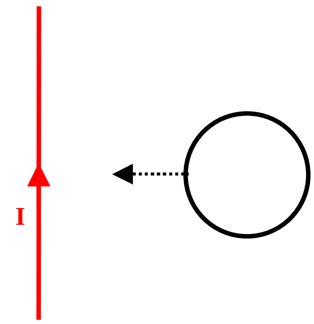
and $I_{tot} = (\pi(3R)^2 - \pi R^2)J = 8\pi R^2 J$, where **J** is the (**uniform**) current density. Hence,

$$B(r) = \frac{2kI_{enclosed}}{r} = \frac{2k(r^2 - R^2)I_{tot}}{8rR^2} .$$

Plugging in **r = 2R** gives **B = 3kI/(8R)**.

Chapter 31 Solutions

Problem 1:



A long straight wire carries current **I**. Nearby and lying in the same plane is a circular loop, as shown in the **Figure**. If the loop is moved toward the wire, what will be the direction of the current induced in the loop (if any) and what will be the direction of any electromagnetic force exerted on the loop?

Answer: counterclockwise; away from the wire

Solution: Pushing the wire loop toward the current carrying wire causes the magnetic flux to increase in the loop and thus the induced current will flow in a **counterclockwise direction** so that the induced magnetic field will be up (opposite to the increasing downward external magnetic field). The force on the loop is in a direction that opposes the push (i.e. **away from the straight wire**).

Problem 2:

A magnetic field given by **B(t) = at+b** with **a = 1 T/s** and **b = -1 T** is directed perpendicular to the plane of a circular coil of **10 turns** and radius **0.2 m**. If the coil's total resistance is **1.58 Ohms**, how much power (**in Watts**) is dissipated at time **t = 1 s**?

Answer: 1

Solution: The magnetic flux through the loop is **Φ_B = NBA** and from **Faraday's Law** we have,

$$\varepsilon = -\frac{d\Phi_B}{dt} = -NA\frac{dB}{dt} = -NAa \quad .$$

At **t = 1 second** (actually at any time t) this gives

$$I = \frac{|\varepsilon|}{R} = \frac{NAa}{R} \, ,$$

and power

$$P = I^2 R = \frac{N^2 A^2 a^2}{R} = \frac{(100)\pi^2 (0.2)^4 (1T/s)^2}{1.58\Omega} = 1W \quad .$$

Problem 3:

A **25-turn** coil of resistance **3 Ohms** has area of **8 cm^2**. Its plane is perpendicular to a magnetic field given by **B(t) = 0.4t - 0.3t^2** (where B is in Tesla and t is in seconds). What is the induced current in the coil (**in milliA**) at **t = 1 second**?

Answer: 1.33

Solution: The magnetic flux through the loop is **Φ_B = NBA** and from **Faraday's Law** we have,

$$\varepsilon = -\frac{d\Phi_B}{dt} = -NA\frac{dB}{dt} = -NA(0.4-0.6t) .$$

At **t = 1 second** this gives

$$I = \frac{|\varepsilon|}{R} = \frac{NA(0.2T/s)}{3\Omega} = \frac{25(8\times10^{-4}A)(0.2T/s)}{3\Omega} = 1.33mA .$$

Problem 4:

A uniform time dependent magnetic field given by **B(t) = at^2** with **a = 0.3 T/s^2** is directed perpendicular to the plane of a square wire loop with sides of length **0.5 m**. If the total resistance of the square wire is **0.4 Ohms**, how much total energy (**in Joules**) is dissipated by heat in the wire during the time from **t = 0** to **t = 10** seconds?

Answer: 18.8

Solution: The magnetic flux through the loop is **Φ_B = BA = BL^2** and from **Faraday's Law** we have,

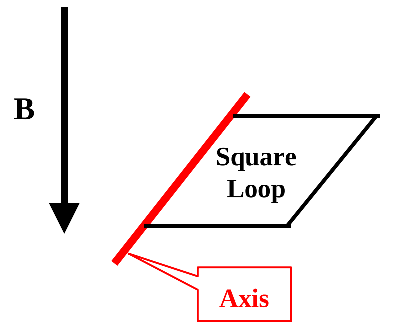
$$\varepsilon = -\frac{d\Phi_B}{dt} = -L^2\frac{dB}{dt} = -2atL^2 .$$

The induced current in the wire is, **I = |ε|/R = 2atL^2/R**, and the power dissipated by heat in the wire is **P = dU/dt = I^{2}R = 4a^{2}t^{2}L^4/R**. Hence,

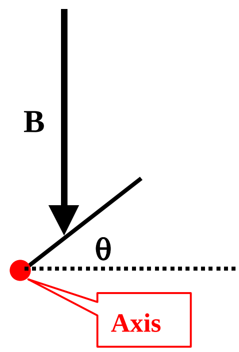
$$U = \int_0^{10s} Pdt = \int_0^{10s} \frac{4a^2t^2L^4}{R}dt = \frac{4a^2(10s)^3L^4}{3R}$$

$$= \frac{4(0.3T/s^2)^2(10s)^3(0.5m)^4}{3(0.4\Omega)} = 18.75J .$$

Problem 5:



A single wire square loop with area **A = 100 cm^2** rotates with a constant angular velocity in the magnetic field of the earth (**about 100 microT**). The axis of rotation goes through one side of the square loop (in the plane of the loop) and is perpendicular to the magnetic field as shown in the Figure. If the period of the rotation is **one second**, what is the maximum EMF generated in the loop (**microV**)?



Answer: 6.3

Solution: The magnetic flux through the loop is **Φ_B = BAcosθ** and from **Faraday's Law** we have,

$$\varepsilon = -\frac{d\Phi_B}{dt} = -BA\frac{d\cos\theta}{dt} = BA\,\omega\sin\theta\ ,$$

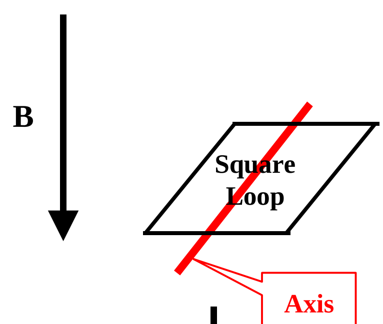
where **ω = dθ/dt** is the angular velocity. The maximum **EMF** occurs when **sinθ = 1** and is given by

$$\varepsilon_{\max} = \omega BA = \frac{2\pi BA}{T}$$

$$= \frac{2\pi(100\times10^{-6}T)(100\times10^{-4}m)}{1s} = 6.28\,\mu V$$

where I used **ω = 2π/T**.

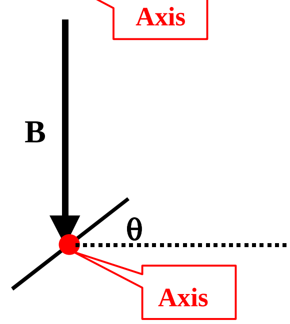
Problem 6:



How will the answer in the previous problem change if one moves the axis of rotation of the loop from the side to the center of the loop (in the plane of the loop) as shown in the **Figure**?

Answer: no change

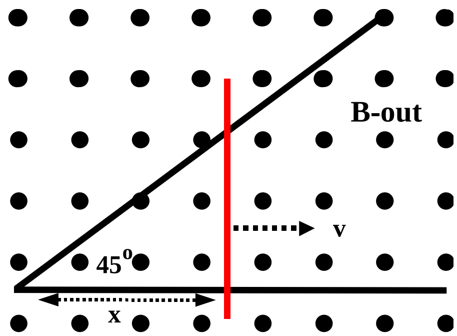
Solution: The magnetic flux through the loop is still given by **Φ_B = BAcosθ** and the induced **EMF** is,



$$\varepsilon = -\frac{d\Phi_B}{dt} = -BA\frac{d\cos\theta}{dt} = BA\,\omega\sin\theta\ ,$$

where **ω = dθ/dt** is the angular velocity. The maximum **EMF** occurs when **sinθ = 1** and is the same as in the previous problem.

Problem 7:



A moveable (massless and frictionless) bar is being moved at a constant velocity of **3 m/s** from left to right along two conducting rails as shown in the **Figure**. The two conducting rails make an angle of **45°** with each other and together with the moving bar form a right triangle. If the system is immersed in a uniform magnetic field (out of the paper) with magnitude **B = 0.5 Tesla** and if the bar was at **x = 0** at **t = 0**, what is the magnitude of the induced EMF in the right triangle (**in Volts**) at **t = 2 seconds**?

Answer: 9

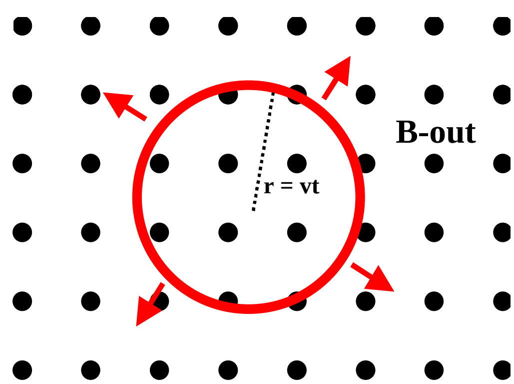
Solution: The magnetic flux through the right triangle is

$$\Phi_B = BA = \frac{1}{2}x^2B = \frac{1}{2}v^2t^2B$$

where I used **A = x²/2** and **x = vt**. Now from **Faraday's Law** we have,

$$|\varepsilon| = \frac{d\Phi_B}{dt} = v^2tB = (3m/s)^2(2s)(0.5T) = 9V\ .$$

Problem 8:



A wire circular loop with a resistance per unit length of **0.5 Ohms/meter** is placed in a uniform magnetic field given by $\vec{B} = B_0\hat{z}$, with $\mathbf{B_0}$ **= 2 Tesla**, as shown in the **Figure** (z is out of the paper). If the radius of the circular loop is increasing at a constant rate given by **r(t) = vt** with **v = 0.5 m/s**, what is the magnitude of the induced EMF (**in Volts**) at **t = 10 seconds**?

Answer: 31.4

Solution: If I choose my orientation to be counterclockwise then the magnetic flux through the loop is $\mathbf{\Phi_B = B_0\pi r^2 = B_0\pi v^2 t^2}$ and from **Faraday's Law** we have,

$$\varepsilon = -\frac{d\Phi_B}{dt} = -2\pi B_0 v^2 t\,,$$

and

$$|\,\varepsilon\,| = 2\pi B_0 v^2 t = 2\pi(2T)(0.5m/s)^2(10s) = 31.4V\,.$$

Problem 9:

In the presious problem, what is the magnitude (**in Amps**) and direction of the induced current in the loop at **t = 10 seconds**?

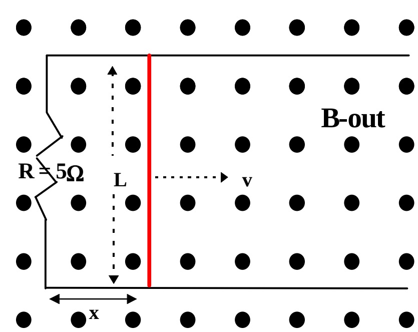
Answer: 2, clockwise

Solution: The magnitude of the induced current is

$$I = \frac{|\,\varepsilon\,|}{R} = \frac{2\pi B_0 r v}{2\pi r(0.5\Omega/m)} = \frac{(2T)(0.5m/s)}{0.5\Omega/m} = 2A\,.$$

The direction of the current is **clockwise** (opposite my orientation).

Problem 10:



A moveable (massless and frictionless) bar with a length of **L = 1** meter is being moved at a constant velocity of **10 m/s** from left to right along two conducting rails by an external force, F, as shown in the Figure. If the system is immersed in a uniform magnetic field (out of the paper) with magnitude **B=2 T**, what is the induced current (**in Amps**) in the **5 Ohm** resistor?

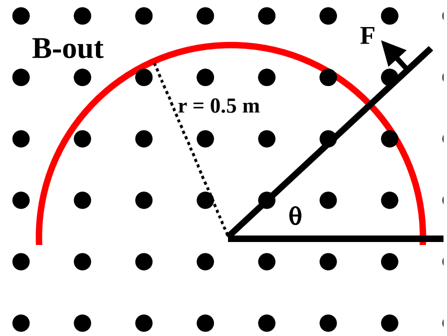
Answer: 4

Solution: If I choose my orientation to be counterclockwise then the magnetic flux through the loop is $\mathbf{\Phi_B = BA = BxL = BLvt}$ and from **Faraday's Law** we have, $\varepsilon = -\frac{d\Phi_B}{dt} = -BLv$ and hence,

$$I = \frac{|\varepsilon|}{R} = \frac{BLv}{R} = \frac{(2T)(1m)(10m/s)}{5\Omega} = 4A\,.$$

Problem 11:

A moveable (massless and frictionless) conducting rod is pulled by an external force so that it rotates with a constant angular velocity along a semicircular loop of wire with radius of **r = 0.5 m** as shown in the **Figure**. The rotating rod is connected at the center of the semicircular loop to a stationary rod that is also in contact with the semicircular loop. If the entire system is placed in a uniform **2 Tesla** magnetic field (**out of the page**), and if it takes **1 second** for the angle between the two rods to go from **$\theta = 0$** to **$\theta = 90^o$**, what is the magnitude of the induced emf (**in Volts**) in the circuit?



Answer: 0.4

Solution: If I choose my orientation to be counterclockwise then the magnetic flux through the circuit is

$$\Phi_B = \frac{1}{2}\theta R^2 B \ ,$$

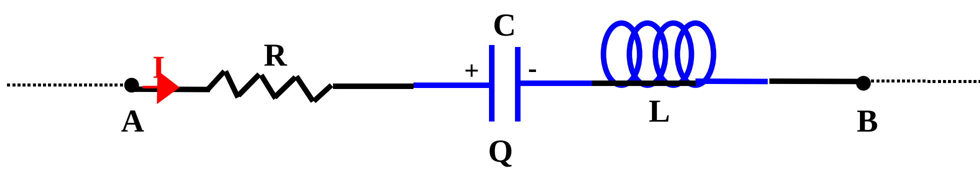
and **Faraday's Law** implies,

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{1}{2}\omega R^2 B \text{ , where } \omega = \frac{d\theta}{dt} \ .$$

Plugging in the numbers yields,

$$|\varepsilon| = \frac{1}{2}\omega R^2 B = \frac{1}{2}\left(\frac{\pi/2}{1s}\right)(0.5m)^2 2T = 0.39V \ ,$$

where I used **$\omega = (\pi/2)/1s$**.



Problem 12:

A current **I** flows through a resistor with **R=25 kΩ**, a capacitor with **C=4 milliF**, and an inductor with **L=7 Henry** as shown in the Figure. At a time when **Q=8 milliC** and the current **I=0.2 milliAmps** and is changing at a rate **dI/dt=1.0 A/s**, what is the potential difference **V_B-V_A** (**inVolts**)?

Answer: -14

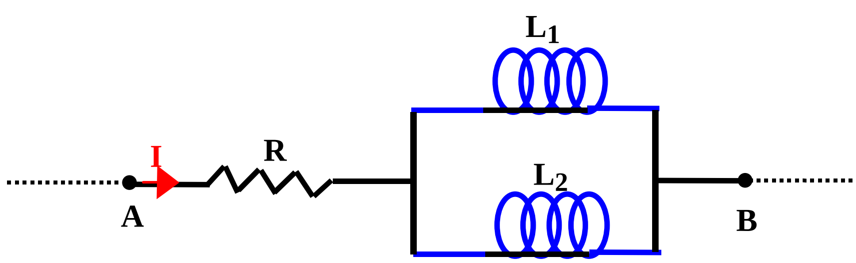
Solution: Moving in the direction of the current flow we have

$$V_B - V_A = -IR - \frac{Q}{C} - L\frac{dI}{dt},$$

and

$$IR = (0.2\times10^{-3}A)(25\times10^{3}\Omega) = 5V$$
$$\frac{Q}{C} = \frac{8mC}{4mF} = 2V$$
$$L\frac{dI}{dt} = (7H)(1A/s) = 7V$$

Thus, V_B-V_A = -5V -2V -7V = -14V.



Problem 13:

A current **I** flows through a resistor with **R = 333 Ω**, and two inductors in parallel with **L_1 = 2 Henry** and **L_2 = 4 Henry** as shown in the **Figure**. At a time when the current **I = 1.0 milliAmps** and is changing at a rate **dI/dt = -1.0 A/s**, what is the potential difference **V_B-V_A** (**in Volts**)?

Answer: 1

Solution: Moving in the direction of the current flow we have

$$V_B - V_A = -IR - L_{eff}\frac{dI}{dt},$$

where

$$\frac{1}{L_{eff}} = \frac{1}{L_1} + \frac{1}{L_2} = \frac{1}{2H} + \frac{1}{4H} = \frac{3}{4H},$$

since for inductors in parallel you add the inverses.
Thus, $\mathbf{L_{eff} = 1.33\ H}$ and

$$-IR = -(1\times10^{-3}A)(333\Omega) = -0.333V$$
$$-L_{eff}\frac{dI}{dt} = -(1.33H)(-1A/s) = 1.33V$$

so that, $\mathbf{V_B - V_A = -0.333V + 1.33V = 1V.}$

Problem 14:

What is the magnitude of the uniform magnetic field (**in Tesla**) that contains as much energy per unit volume as a uniform **3,000 V/m** electric field?

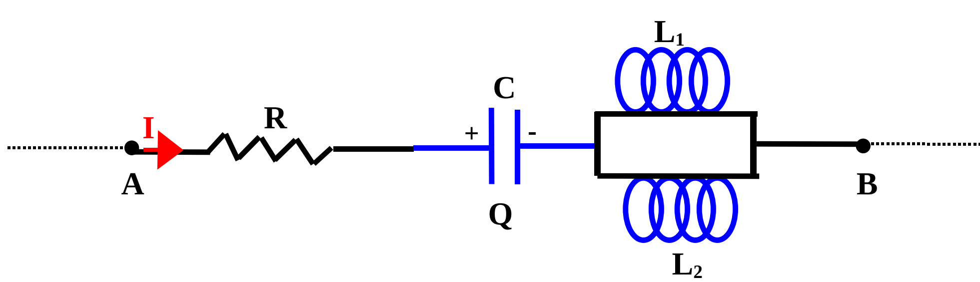
Answer: $1\mathrm{x}10^{-5}$

Solution: Setting the magnetic field energy density equal to the electric field energy density yields

$$u_B = \frac{1}{2\mu_0}B^2 = u_E = \frac{1}{2}\varepsilon_0 E^2$$

and solving for **B** gives

$$B = \sqrt{\mu_0\varepsilon_0}E = \frac{E}{c} = \frac{3{,}000\,V/m}{3\times10^8\,m/s} = 1\times10^{-5}T.$$



Problem 15:

A current **I** flows through a resistor with **R = 2 Ω**, a capacitor with **C = 3 milliF**, and and two inductors in parallel (**L_1 = 1 milliHenry** and **L_2 = 3 milliHenry**) as shown in the **Figure**. At a time when **Q = 0.6 microC** and the current **I = 0.2 milliAmps** and is decreasing at a rate of **0.8 Amps/s**, what is the potential difference **V_B-V_A (milliVolts)**?

Answer: zero

Solution: Moving in the direction of the current flow we have

$$V_B - V_A = -IR - \frac{Q}{C} - L_{eff}\frac{dI}{dt},$$

where

$$\frac{1}{L_{eff}} = \frac{1}{L_1} + \frac{1}{L_2} \quad \text{or} \quad L_{eff} = \frac{L_1 L_2}{L_1 + L_2} = \frac{(1mH)(3mH)}{1mH + 3mH} = \frac{3}{4}mH$$

and

$$-IR = -(0.2mA)(2\Omega) = -0.4mV$$
$$-\frac{Q}{C} = -\frac{0.6\mu C}{3mF} = -0.2mV$$
$$-L_{eff}\frac{dI}{dt} = -\frac{3}{4}mH(-0.8A/s) = +0.6mV$$

Thus, **V_B-V_A = -0.4 mV –0.2 mV + 0.6 mV = 0**.

Problem 16:

In an **LR circuit** where **L = 120 milliH** and **R= 15 ohms**, if the switch is closed at t = 0 how long (**in milli-seconds**) does it take for the current to reach **75%** of its final value?

Answer: 11.1

Solution: The current is given by

$$I(t) = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau}\right) = I_{\max}\left(1 - e^{-t/\tau}\right)$$

and thus

$$e^{-t/\tau} = 1 - I/I_{\max}$$

and solving for **t** gives

$$t = -\tau \ln\left(1 - I/I_{\max}\right) = -(8ms)\ln\left(1 - 0.75\right) = 11.1ms,$$

where I have used **τ = L/R = 8ms**.

Problem 17:

In an LR circuit where **L = 120 milliH** and **R = 15 ohms**, if the switch is closed at **t = 0** how long (**in milli-seconds**) does it take for the stored energy in the inductor to reach **50%** of its maximum value?

Answer: 5.5

Solution: The current and the stored energy are given by

$$I(t) = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau}\right)$$

$$U(t) = \frac{1}{2}LI^2 = U_{\max}\left(1 - e^{-t/\tau}\right)^2$$

and thus

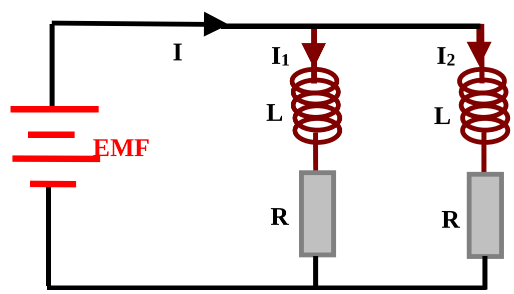
$$e^{-t/\tau} = 1 - \sqrt{U/U_{\max}}$$

and solving for **t** gives

$$t = -\tau \ln\left(1 - \sqrt{U/U_{\max}}\right) = -(8ms)\ln\left(1 - \sqrt{0.5}\right) = 9.8ms\,,$$

where I have used **τ = L/R = 8ms**.

Problem 18:



Consider the **LR circuit** shown in the **Figure** which consists of two equal inductors, **L**, and two equal resistors, **R**, and an **EMF**. If **L = 4 milliHenry** and **R= 2 milliOhms** and if the switch is closed at **t = 0**, how long (**in seconds**) does it take for the total current in the circuit to reach **90%** of its final value?

Answer: 4.6

Solution: Using the **"Loop Rule"** for circuits we see that

$$\varepsilon - L\frac{dI_1}{dt} - I_1R = 0$$

$$\varepsilon - L\frac{dI_2}{dt} - I_2R = 0$$

and adding these two equations yields

$$2\varepsilon - L\frac{dI}{dt} - IR = 0\,,$$

where the total current **I = I$_1$ + I$_2$**. The above equation for **I** is equivalent to the following

$$\varepsilon - L_{eff}\frac{dI}{dt} - IR_{eff} = 0\ ,$$

where **L$_{eff}$ = L/2** and **R$_{eff}$ = R/2** and we know the solution is given by

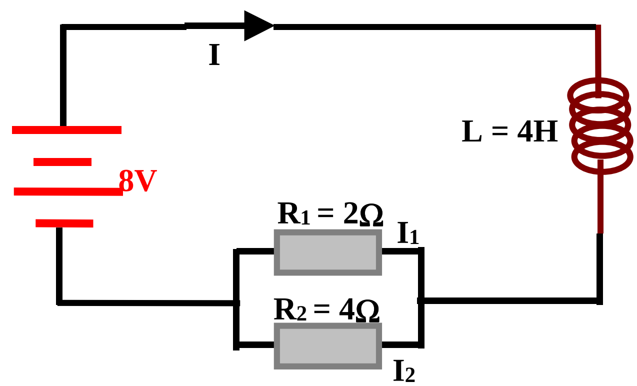
$$I(t) = I_{\max}\left(1 - e^{-t/\tau}\right) = \frac{\varepsilon}{R_{eff}}\left(1 - e^{-t/\tau}\right),$$

with **t = L$_{eff}$/R$_{eff}$ = L/R = (4 mH)/(2 mΩ) = 2 s**. Solving for the time gives

$$t = -\tau \ln\left(1 - I(t)/I_{\max}\right)$$
$$= -(2s)\ln(1-0.9) = 4.605\,s$$

Problem 19:

Consider the LR circuit shown in the **Figure** which consists a **8 Volt** EMF, an inductor, **L = 4 H**, and two resistors. If the switch is closed at **t = 0**, when (**in seconds**) is the current through the **2 Ω** resistor equal to **1 Amp**?



Answer: 0.86

Solution: The total current in the circuit, **I = I$_1$ + I$_2$**, is given by

$$I(t) = I_{\max}\left(1 - e^{-t/\tau}\right) = \frac{\varepsilon}{R_{eff}}\left(1 - e^{-t/\tau}\right),$$

where **τ = L/R$_{eff}$** and

$$\frac{1}{R_{eff}} = \frac{1}{R_1} + \frac{1}{R_2} \quad \text{or} \quad R_{eff} = \frac{R_1R_2}{R_1+R_2} = \frac{(2\Omega)(4\Omega)}{6\Omega} = \frac{4}{3}\Omega\ .$$

The current **I$_1$** is given by

$$I_1(t) = \frac{R_2}{R_1+R_2}I(t) = \frac{\varepsilon}{R_1}\left(1 - e^{-t/\tau}\right),$$

and solving for **t** yields

$$t = -\tau \ln\left(1 - \frac{R_1 I_1}{\varepsilon}\right) = -\left(\frac{4H}{4/3\Omega}\right)\ln\left(1 - \frac{(2\Omega)(1A)}{8V}\right)$$

$$= -(3s)\ln\left(1 - \frac{1}{4}\right) = -(3s)\ln(0.75) = 0.863\,s$$

Chapter 32 Solutions

Problem 1:

A parallel-plate capacitor whose plates have radius **R** is being charged by a current **I**. What is the magnitude of the magnetic field (**in Tesla**) between the plates a distance **r = R/2** from the center?

Answer: kI/R

Solution: We know from **Ampere's Law (with J = 0 inside the capacitor)** that

$$\oint \vec{B}\cdot d\vec{l} = \mu_0\varepsilon_0\frac{d\Phi_E}{dt}.$$

If I choose a loop of radius **r** within the capacitor then $\mathbf{\Phi_E = E\pi r^2}$ and

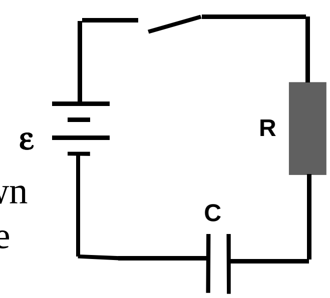
$$2\pi r B(r) = \mu_0\varepsilon_0\pi r^2\frac{dE}{dt} = \mu_0\varepsilon_0\pi r^2\left(\frac{I}{\varepsilon_0\pi R^2}\right),$$

where I have used $\mathbf{E = Q/(\varepsilon_0 A)}$, $\mathbf{A = \pi R^2}$, and **I = dQ/dt**. Solving for **B** gives

$$B(r) = \frac{2kIr}{R^2},$$

which for **r = R/2** becomes **B = kI/R.**

Problem 2:



A parallel-plate capacitor whose plates have radius **r** is being charged through a resistor **R** by an EMF **ε** as shown in the **Figure**. If the switch is closed at **t = 0**, what is the maximum magnitude of the magnetic field between the plates of the capacitor a distance **r/2** from the center? (Note: $k = \mu_0/4\pi$)

Answer: kε/(rR)

Solution: We know from **Ampere's Law (with J = 0 inside the capacitor)** that

$$\oint \vec{B}\cdot d\vec{l} = \mu_0\varepsilon_0\frac{d\Phi_E}{dt}.$$

If I choose a loop of radius $\mathbf{r_a}$ within the capacitor then $\mathbf{\Phi_E = E\pi r_a^2}$ and

$$2\pi r_a B(r_a) = \mu_0\varepsilon_0\pi r_a^2\frac{dE}{dt} = \mu_0\varepsilon_0\pi r_a^2\left(\frac{I}{\varepsilon_0\pi r^2}\right),$$

where I have used $\mathbf{E = Q/(\varepsilon_0 A)}$, $\mathbf{A = \pi r^2}$, and $\mathbf{I = dQ/dt}$. The maximum magnetic field occurs when the current is maximum and solving for $\mathbf{B_{max}}$ at $\mathbf{r_a = r/2}$ gives

$$B_{\max} = \frac{kI_{\max}}{r} = \frac{k\varepsilon}{rR},$$

where I used $\mathbf{I_{max} = \varepsilon/R}$ (which occurs at t = 0).

Problem 3:

The potential difference, $\mathbf{V_C}$, across a **3 Farad** parallel-plate capacitor whose plates have radius **R = 2 cm** varies with time and is given by $\mathbf{V_C(t) = at}$, with **a = 10 V/s**. What is the magnitude of the magnetic field (**in microT**) between the plates of the capacitor a distance **R** from the center?

Answer: 300

Solution: We know from **Ampere's Law (**with J = 0 inside the capacitor**)** that

$$\oint \vec{B}\cdot d\vec{l} = \mu_0\varepsilon_0\frac{d\Phi_E}{dt}.$$

If I choose a loop of radius **R** within the capacitor then $\mathbf{\Phi_E = E\pi R^2}$ and

$$2\pi RB(R) = \mu_0\varepsilon_0\pi R^2\frac{dE}{dt} = \mu_0\varepsilon_0\pi R^2\left(\frac{I}{\varepsilon_0\pi R^2}\right),$$

where I have used $\mathbf{E = Q/(\varepsilon_0 A)}$, $\mathbf{A = \pi R^2}$, and $\mathbf{I = dQ/dt}$. Solving for **B** gives

$$B(R) = \frac{2k}{R}\frac{dQ}{dt} = \frac{2k}{R}C\frac{dV}{dt}$$
$$= \frac{2(10^{-7}Tm/A)(3F)(10V/s)}{2\times10^{-2}m} = 300\,\mu V$$

where I used **Q = CV** which implies that **dQ/dt = C dV/dt** with **dV/dt = a**.

Problem 4:

A **2 milliFarad** parallel-plate capacitor whose plates are circular with radius **R = 0.5 cm** is connected to an EMF-source that increases uniformly with time at a rate of **150 V/s**. What is the magnitude of the magnetic **field (in microTesla)** between the plates of the capacitor a distance **r = R** from the center?

Answer: 12

Solution: We know from **Ampere's Law (with J = 0 inside the capacitor)** that

$$\oint \vec{B}\cdot d\vec{l} = \mu_0\varepsilon_0 \frac{d\Phi_E}{dt}.$$

If I choose a loop of radius **R** around the capacitor then $\mathbf{\Phi_E = E\pi R^2}$ and the potential difference across the capacitor is **V = Ed**, where **d** is the seperation of the plates. Hence,

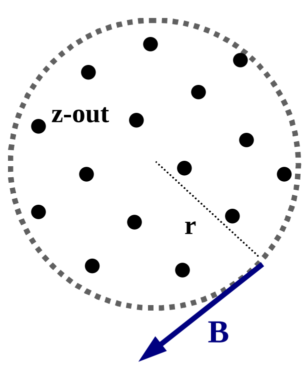
$$2\pi RB(R) = \mu_0\varepsilon_0\pi R^2\frac{dE}{dt} = \mu_0\varepsilon_0\pi R^2\left(\frac{1}{d}\frac{dV}{dt}\right) = \mu_0 C\frac{dV}{dt},$$

where I have used $\mathbf{C = \varepsilon_0 A/d}$ with $\mathbf{A = \pi R^2}$. Solving for **B** gives

$$B = \frac{2k}{R}C\frac{dV}{dt}$$
$$= \frac{2(10^{-7}Tm/A)(2\times10^{-3}F)(150V/s)}{0.5\times10^{-2}m} = 12\mu T.$$

Problem 5:

Consider the situation illustrated in the figure. A uniform electric field points in the **z direction** *(out of the paper)* with a value given by $\mathbf{E_z(t)=a - bt}$, where **a = 18 V/m** and **b = 9 V/(m s)** and is confined to a circular area with a radius **R = 4 meters**. What is the magnitude and direction of the magnetic field (**in picoTesla**) at a point **P** on the circumference of the circle at the time **t = 2 seconds**?



Answer: $\mathbf{2x10^{-4}}$, clockwise

Solution: We know from **Ampere's Law (with J = 0)** that

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} .$$

If I choose my orientation to by counter-clockwise then $\mathbf{\Phi_E = E\pi r^2}$ and

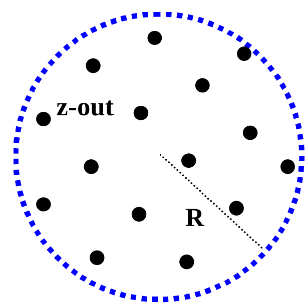
$$2\pi r B(r) = \mu_0 \varepsilon_0 \pi r^2 \frac{dE}{dt} = -\frac{\pi r^2 b}{c^2} ,$$

and solving for **B** gives

$$B(r) = -\frac{rb}{2c^2} = -\frac{(4m)(9V/ms)}{2(3\times 10^8 m/s)^2} = -2\times 10^{-4}\, pT .$$

Since **B** is negative it points opposite the direction of my orientation (**clockwise**).

Problem 6:



Consider the situation illustrated in the **Figure**. A uniform electric field points in the z direction (*out of the paper*) with a value given by $\mathbf{E_z(t) = A\sin\omega t}$, where $\mathbf{A = 90\ V/m}$ and $\mathbf{\omega = 1{,}000\ radians/sec}$ and is confined to a circular area with a radius $\mathbf{R = 2\ meters}$. What is the maximum magnitude of the magnetic field (**in picoTesla**) at a point P on the circumference of the circle?

Answer: 1

Solution: We know from **Ampere's Law (with J = 0)** that

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} .$$

If I choose my orientation to by counter-clockwise then $\mathbf{\Phi_E = E\pi R^2}$ and

$$2\pi R B(R) = \mu_0 \varepsilon_0 \pi R^2 \frac{dE}{dt} = \mu_0 \varepsilon_0 \pi R^2 A\omega \cos(\omega t) ,$$

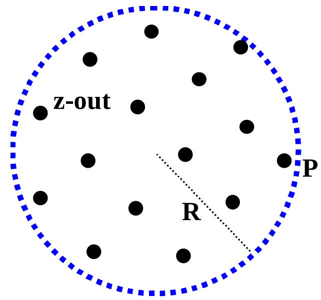
and solving for **B** gives

$$B(R) = \frac{RA\omega}{2c^2} \cos(\omega t) ,$$

and hence

$$B_{\max} = \frac{RA\omega}{2c^2} = \frac{(2m)(90V/m)(1{,}000/s)}{2(3\times 10^8 m/s)^2} = 1\, pT .$$

Problem 7:



Consider the situation illustrated in the **Figure**. A uniform electric field points in the z-direction (out of the paper) with a value given by $\mathbf{E_z(t) = at}$, and a uniform magnetic field points in the z-direction with a value given by $\mathbf{B_z(t) = bt}$. Both fields are confined to a circular region with a radius **R**. If **a = 1 V/(m s)** and **b = 1 T/s** and if the magnitude of the ***induced*** electric field at the point **P** on the circumference of the circle at the time **t = 2 seconds** is **9,000 V/m**, what is the magnitude ***induced*** magnetic field (**in picoTesla**) at the point **P** at the time **t = 2 seconds**?

Answer: 0.1

Solution: We know from **Ampere's Law (**with $\mathbf{I_{enclosed} = 0}$**)** that

$$\oint \vec{B}\cdot d\vec{l} = \mu_0\varepsilon_0 \frac{d\Phi_E}{dt} .$$

If I choose my orientation to by counter-clockwise then $\mathbf{\Phi_E = E\pi R^2}$ and

$$2\pi R B(R) = \mu_0\varepsilon_0 \pi R^2 \frac{dE}{dt} = \frac{\pi R^2 a}{c^2} ,$$

and solving for the **induced magnetic field B** gives

$$B(R) = \frac{Ra}{2c^2} .$$

Now from **Faraday's Law** we know that

$$\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt} .$$

If I choose my orientation to by counter-clockwise then $\mathbf{\Phi_B = B\pi R^2}$ and

$$2\pi R E(R) = -\pi R^2 \frac{dB}{dt} = -\pi R^2 b ,$$

and solving for the **magnitude** of the **induced electric field E** gives

$$E(R) = \frac{Rb}{2} .$$

Thus,

$$B(R) = \frac{aE(R)}{bc^2} = \frac{(1V/ms)(9{,}000\,V/m)}{(1T/s)(3\times 10^8\,m/s)^2} = 0.1\,pT .$$

Chapter 33 Solutions

Problem 1:

If the total energy in an **LC** circuit is **5.0 microJoules** and **L=25 milliH**, then what is the maximum current (**in milliA**)?

Answer: 20

Solution: The maximum stored energy in the inductor is

$$U_{\max} = \frac{1}{2} L I_{\max}^2$$

and solving for $\mathbf{I_{max}}$ yields

$$I_{\max} = \sqrt{\frac{2U_{\max}}{L}} = \sqrt{\frac{2(5\mu J)}{25mH}} = 20mA.$$

Problem 2:

A **10 microF** capacitor with an initial charge $\mathbf{Q_0}$ is connected across **an 8 milliH** inductor. If the circuit is completed at time t = 0, how soon (**in millisec**) afterward will the charge on the capacitor reverse sign and become equal to $\mathbf{-Q_0}$?

Answer: 0.89

Solution: For this **LC** circuit $\mathbf{Q = Q_0 \cos(\omega t)}$ which is equal to $\mathbf{-Q_0}$ when $\mathbf{\omega t = \pi}$ so that

$$t = \frac{\pi}{\omega} = \pi\sqrt{LC} = \pi\sqrt{(8mH)(10\mu F)} = 0.89ms.$$

Problem 3:

A charged capacitor is connected across an inductor to form an LC circuit. When the charge on the capacitor is **0.1 C** the current is **0.5 Amps**. If it the period of the LC oscillations is **2 seconds** what is the maximum charge on the capacitor (**in C**)?

Answer: 0.19

Solution: We know that at some time (say t = 0) that $\mathbf{Q_0 = 0.1}$ **C** and $\mathbf{I_0 = 0.5}$ **A**. Energy is consrved in an **LC** circuit which implies that at any other time t,

$$\frac{Q^2}{2C}+\frac{1}{2}LI^2=\frac{Q_0^2}{2C}+\frac{1}{2}LI_0^2 .$$

The maximum charge occurs when I = 0 and hence,

$$\frac{Q_{\max}^2}{2C}=\frac{Q_0^2}{2C}+\frac{1}{2}LI_0^2 .$$

Solving for **Q_{max}** gives

$$Q_{\max}=\sqrt{Q_0^2+(I_0^2/\omega^2)}=\sqrt{Q_0^2+(TI_0/(2\pi))^2}$$
$$=\sqrt{(0.1C)^2+((2s)(0.5A)/(2\pi))^2}=0.188\,C$$

where I used

$$\omega=\frac{1}{\sqrt{LC}}=\frac{2\pi}{T},$$

where T is the period of the oscillations.

Problem 4:

A current flows in an LC circuit with **L = 50 milliH** and **C = 4 microF**. Starting from when the current is maximum, how long will it take (**in millisec**) for the capactor to become fully charged?

Answer: 0.7

Solution: When the current is maximum the charge on the capacitor is zero and it will take one-quarter of a period for it to become fully charged. Thus,

$$t=\frac{T}{4}=\frac{\pi}{2}\sqrt{LC}=\frac{\pi}{2}\sqrt{(50\times10^{-3}H)(4\times10^{-6}F)}=0.7\,ms ,$$

where I used **T = 2π/ω** and $\omega=1/\sqrt{LC}$.

Problem 5:

A **1 milliFarad** capacitor with an initial charge **Q_0** is connected across a **9 milliHenry** inductor to form an **LC circuit**. If the circuit is completed at time **t = 0**, how soon (**in millisec**) afterward will the charge on the capacitor decrease to one-half of its initial value?

Answer: 3.14

Solution: We know that

$$Q(t) = Q_0 \cos(\omega t)$$

where $\omega = 1/\sqrt{LC}$. Solving for **t** gives

$$t = \cos^{-1}(Q/Q_0)/\omega = \sqrt{LC}\cos^{-1}(Q/Q_0)$$
$$= \sqrt{(9\times10^{-3}H)(1\times10^{-3}F)}\cos^{-1}(0.5)$$
$$= (3ms)(\pi/3) = 3.14\,ms$$

Problem 6:

In the previous problem, if the maximum current is **3 milliAmps** what is $\mathbf{Q_0}$ (**in microC**)?

Answer: 9

Solution: We know that the amplitude of the current oscillations, $\mathbf{I_{max}}$, is given by $\mathbf{I_{max} = Q_0}\omega$, where $\omega = 1/\sqrt{LC}$. Hence,

$$Q_0 = \frac{I_{max}}{\omega} = (3\times10^{-3}A)(3ms) = 9\times10^{-6}C \,.$$

Problem 7:

A capacitor (**C=8 milliFarads**) initially charged by connecting it to a **2 Volt** battery is connected to an inductor (**L=2 milliHenry**) to form an LC circuit (the resistance of the circuit is negligible). During subsequent oscillations, what is the maximum energy stored in the magnetic field of the inductor (**in milliJoules**)?

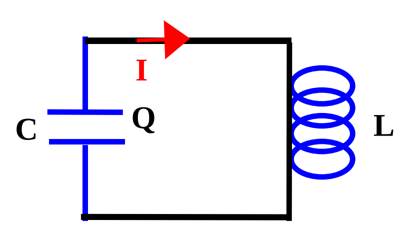
Answer: 16

Solution: Since energy is conserved the maximum stored energy in the inductor is eual to the initial stored energy as follows:

$$U_{max} = U_0 = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}(8mF)(2V)^2 = 16mJ \,.$$

Problem 8:

A capacitor **C** and an inductor **L** form an **LC circuit** as shown in the **Figure**. At a certain instant of time when the current **I = 2 Amps** the charge on the capacitor is **Q = 0.01 C**. A short



time later when the current **I = 1 Amps** the charge on the capacitor is **Q = 0.02 C**. What is the frequency of the oscillations (**in Hz**)?

Answer: 15.9

Solution: From **energy conservation** we know that

$$\frac{Q_1^2}{2C} + \frac{1}{2} L I_1^2 = \frac{Q_2^2}{2C} + \frac{1}{2} L I_2^2$$

and multiplying by **2/L** gives

$$\omega^2 Q_1^2 + I_1^2 = \omega^2 Q_2^2 + I_2^2 \, ,$$

where $\omega = 1/\sqrt{LC}$. Solving for **ω** yields

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{I_2^2 - I_1^2}{Q_1^2 - Q_2^2}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{(1A)^2 - (2A)^2}{(0.01C)^2 - (0.02C)^2}} = \frac{100}{2\pi} = 15.9 Hz$$

Chapter 34 Solutions

Problem 1:

The electric field component of an electromagnetic wave traveling in a vacuum is given by **$E_y = E_0 \sin(kx - \omega t)$**, where **$E_0$ = 300 V/m** and **$k=10^7$/m**. What are the frequency of the oscillations (**in Hz**) and the direction of propagation?

Answer: $4.77\text{x}10^{14}$, +x direction

Solution: The angular frequency **$\omega = 2\pi f$** and the speed of light in a vacuum is **$c = \omega/k$**. Hence,

$$f = \frac{\omega}{2\pi} = \frac{ck}{2\pi} = \frac{(3\times 10^8 m/s)(10^7/m)}{2\pi} = 4.77\times 10^{14} Hz \,.$$

Problem 2:

What is the value of the **z-component** of the magnetic field (**in microT**) of the electromagnetic wave in the previous problem at **x = 0** and **t = $5.25\text{x}10^{-16}$** seconds?

Answer: -1

Solution: The z-component of the magnetic field is given by

$$B_z = \frac{E_0}{c}\sin(kx - \omega t) = -\frac{E_0}{c} = -\frac{300 V/m}{3\times 10^8 m/s} = -1\mu T \,,$$

where I have used

$$\sin(kx - \omega t) = -\sin(\omega t) = -\sin(2\pi f t) = -\sin(\pi/2) = -1 \,,$$

since **$ft = (4.77\text{x}10^{14}/s)(5.25\text{x}10^{-16}s) = 0.25$**.

Problem 3:

The electric field component of an electromagnetic plane wave traveling in a vacuum is given by $\vec{E} = E_0 \sin(kx + \omega t)\hat{y}$, where **$E_0$ = 300 V/m** and **k = $1.25\text{x}10^7$/m**. What is the magnetic field component of the electromagnetic wave?

Answer: $\vec{B} = -(E_0/c)\sin(kx + \omega t)\hat{z}$

Solution: We know that **sin(kx+ωt)** corresponds to a wave traveling in the **–x** direction and we know that the direction of propagation of an electromagnetic wave is given by $\vec{E}\times\vec{B}$ and hence,

$$\vec{B} = -(E_0/c)\sin(kx+\omega t)\hat{z},$$

where I also used **B = E/c**.

Problem 4:

What is the frequency of the electromagnetic wave in the previous problem (**in Hz**)?

Answer: 6x10[14]

Solution: We know that **c = ω/k** and **ω = 2πf**. Hence,

$$f = \frac{kc}{2\pi} = \frac{(1.25\times 10^{7}/m)(3\times 10^{8}m/s)}{2\pi} = 6\times 10^{14}\,Hz.$$

Problem 5:

How much force (**in microN**) will the electromagnetic wave in the previous problem exert on one square meter of a wall when it hits the wall (*normal to the surface*) and is completely reflected?

Answer: 0.8

Solution: For total reflection we have

$$F = \frac{2IA}{c} = \frac{2A}{c}\left(\frac{E_0^2}{2\mu_0 c}\right) = \frac{AE_0^2}{\mu_0 c^2}$$
$$= \frac{(1m^2)(300V/m)^2}{(4\pi\times 10^{-7}Tm/A)(3\times 10^{8}m/s)^2} = 0.796\,\mu N$$

Problem 6:

If the amplitude of the electric field oscillations **6 meters** from a point source of electromagnetic radiation is **10 V/m**, what is the power output of the source (**in Watts**)?

Answer: 60

Solution: We know

$$I = \frac{P}{4\pi r^2} = \frac{E_0^2}{2\mu_0 c}$$, where P is the power of the source. Solving for P gives

$$P = \frac{4\pi r^2 E_0^2}{2\mu_0 c} = \frac{4\pi(6m)^2(10V/m)^2}{2(4\pi\times10^{-7}Tm/A)(3\times10^8 m/s)} = 60W \;.$$

Problem 7:

What is the amplitude of the electric field oscillations (**in V/m**) **2 meters** from a **60 Watt** point source of electromagnetic radiation (*assume the radiation is a plane wave*)?

Answer: 30

Solution: We know

$$I = \frac{P}{A} = \frac{E_0^2}{2\mu_0 c}$$, where **P** is the power of the source and where $\mathbf{A = 4\pi r^2}$.

Solving for $\mathbf{E_0}$ gives

$$E_0 = \sqrt{\frac{2\mu_0 cP}{A}} = \sqrt{\frac{2(4\pi\times10^{-7}Tm/A)(3\times10^8 m/s)(60W)}{4\pi(2m)^2}} = 30V/m \;.$$

Problem 8:

Light from a distant point source of electromagnetic radiation exerts a radiation pressure of **one microN per meter squared** when it is completely absorbed by the surface S. What is the maximum value of the magnetic field (**in microT**) within the radiation at S?

Answer: 1.6

Solution: We know

$$P = \frac{F}{A} = \frac{I}{c} = \frac{E_0^2}{2\mu_0 c^2} = \frac{B_0^2}{2\mu_0}$$, where P is the radiation pressure and where I have used **E = cB**. Solving for B_0 gives

$$B_0 = \sqrt{2\mu_0 P} = \sqrt{2(4\pi\times10^{-7}Tm/A)(1\mu N/m^2)} = 1.6\mu T \;.$$

Problem 9:

Light with an intensity of **1.2 kW/m²** falls normally on a surface with an area of **1 m²** and is completely **reflected**. The force of the radiation on the surface (**in microN**) is

Answer: 8

Solution: We know that

$$F = \frac{2IA}{c} = \frac{2(1.2\times 10^{3} W/m^{2})(1m^{2})}{3\times 10^{8} m/s} = 8\mu N \ .$$

Problem 10:

A radio station delivers **75 MegaWatts** of power which is distributed uniformly in all directions to the radio waves. What is the radiation pressure exerted by these waves (**in MicroPascals**) upon a flat, perfect absorber placed perpendicular to these waves a distance of **10 meters** from the transmitter? (Note: **1 Pa = 1 N/m²**)

Answer: 199

Solution: We know that for absorption,

$$P_{\Pr essure} = \frac{I}{c} = \frac{P_{Power}}{4\pi r^{2} c} = \frac{75\times 106\, W}{4\pi(10\, m)^{2}(3\times 10^{8} m/s)} = 199\ \mu Pa \ .$$

Problem 11:

The sun has a mass of **1.99x10³⁰ kg** and a radiation power of **3.9x10²⁶ Watts**. What is the maximum solar radiation force (**in N**) on a perfectly reflecting square piece of foil with sides of length **L = 1 meter** located **one million miles** from the sun (Note: 1 mile = 1,609 meters)?

Answer: 0.08

Solution: The **intensity** of the solar radiation a distance **R** from the sun is given by

$$I_{sun} = \frac{P_{sun}}{4\pi R^{2}} \ ,$$

and the radiation **force** for total reflection (**normal to the surface**) is

$$F = \frac{2I_{sun}}{c}A = \frac{2P_{sun}}{4\pi R^2 c}L^2$$
$$= \frac{2(3.9\times10^{26}W)(1m)^2}{4\pi(1{,}609\times10^6 m)^2(3\times10^8 m/s)} = 0.08N$$

Problem 12:

The sun has a mass of **1.99x10^{30} kg** and a radiation power of **3.90x10^{26} Watts**. A spaceship might be propelled in our solar system by the radiation pressure from the sun, using a large sail made of foil (sailing on the solar wind). If the ship has a mass of **1,000 kg** and if the sail is a perfectly reflecting square with sides of length **L**, what is the smallest value of **L** (**in meters**) that will allow the ship to escape the sun's gravitational attraction? (The gravitational constant **G = 6.67x10^{-11} Nm^2/kg^2**.)

Answer: 801

Solution: To find the minimum sail size we set the magnitude radiation force equal to the magnitude of the gravitational force as follows

$$F_{radiation} = \frac{2I}{c}L^2 = \frac{2P_{sun}}{4\pi r^2 c}L^2 = G\frac{mM_{sun}}{r^2} = F_{gravity} \,,$$

where I have used $\mathbf{I = P_{sun}/(4\pi r^2)}$ with **r** is the distance to the sun. Solving for **L** yields,

$$L = \sqrt{\frac{GmM_{sun}2\pi c}{P_{sun}}}$$
$$= \sqrt{\frac{(6.67\times10^{-11}Nm^2/kg^2)(1{,}000kg)(1.99\times10^{30}kg)2\pi(3\times10^8 m/s)}{3.9\times10^{26}W}} .$$
$$= 801m$$

Problem 13:

A point source of light is located **10 meters** below the surface of a large lake (**n=1.3**). What is the diameter (**in m**) of the largest circle on the pool's surface through which light coming directly from the source can emerge?

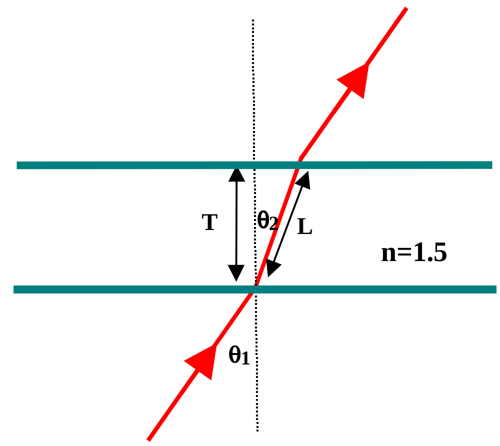
Answer: 24.1

Solution: The critical angle (angle of total internal reflection) is given by $\sin\theta_c = 1/n$ and $\tan\theta_c = R/d$. Thus,

$$D = 2R = 2d\tan\theta_c = \frac{2d}{\sqrt{n^2-1}} = \frac{2(10\text{m})}{\sqrt{(1.3)^2-1}} = 24.1m\ .$$

Problem 14:

A beam of light strikes the surface of a **2-cm** thick parallel-sided sheet of glass (**n = 1.5**) at an incident angle of **35 degrees**. What is the beams actual path length (**in cm**) through the glass?



Answer: 2.16

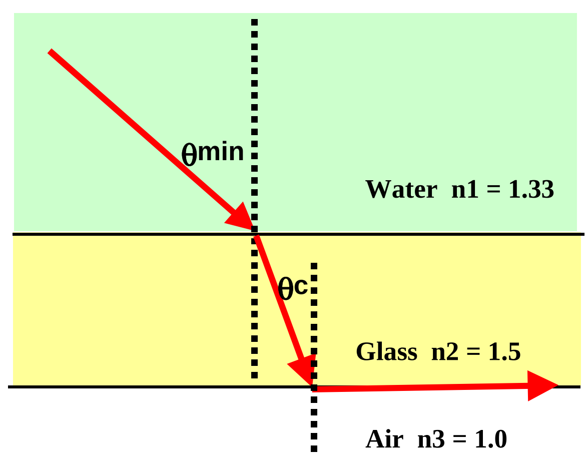
Solution: We know that $n_1\sin\theta_1 = n_2\sin\theta_2$ and $\cos\theta_2 = T/L$, where **T** is the thickness of the glass and **L** is the path length. Thus,

$$L = \frac{T}{\cos\theta_2} = \frac{Tn_2}{\sqrt{n_2^2-\sin^2\theta_1}} = \frac{(2cm)(1.5)}{\sqrt{(1.5)^2-(0.574)^2}} = 2.16cm\ ,$$

where I used $n_1 = 1$ and $\theta_1 = 35^o$.

Problem 15:

A beam of light travelling through water (**n=1.33**) strikes a piece of glass (**n=1.5**) at an incident angle **θ** as shown in the **Figure**. What is the minimum angle **θ** (**in degrees**) such that the light will not reach the air (**n=1**) on the other side of the glass?



Answer: 48.7

Solution: The critical angle (total internal reflection in the glass) is given by

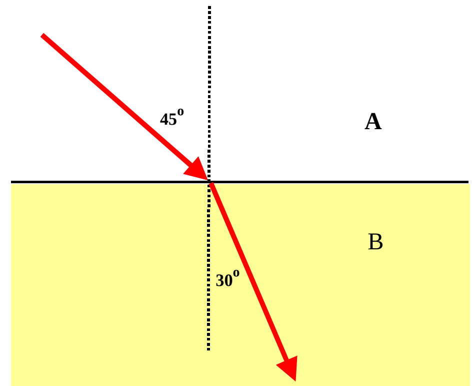
$$n_1 \sin\theta_{\min} = n_2 \sin\theta_c = n_3 \sin(90^\circ)$$

Solving for θ_{min} gives

$$\sin\theta_{\min} = \frac{n_3}{n_1} = \frac{1}{1.33} = 0.75$$

and thus θ_{min} **= 48.7°**.

Problem 16:



A beam of light travelling through medium A at a speed of **2.25x10^8 m/s** strikes medium B at an incident angle of **45 degrees** as shown in the **Figure**. If the refracted angle of the light is **30 degrees**, what is the speed of the light in medium B (**in m/s**)?

Answer: 1.59x10^8

Solution: From Snell's law we have $n_A \sin\theta_A = n_B \sin\theta_B$ and since $n_A = c/v_A$ and $n_B = c/v_B$ this implies

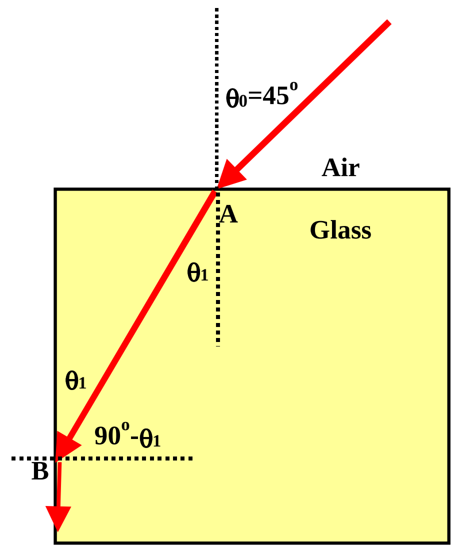
$$\frac{\sin\theta_A}{v_A} = \frac{\sin\theta_B}{v_B}.$$

Solving for v_B gives

$$v_B = \frac{\sin\theta_B}{\sin\theta_A} v_A$$

$$= \frac{\sin(30^\circ)}{\sin(45^\circ)}(2.25\times 10^8 m/s) = 1.59\times 10^8 m/s$$

Problem 17:



A light ray enters a rectangular glass slab at point **A** at an incident angle of **45 degrees** and then undergoes total internal reflection at point **B** as shown in the **Figure**. What minimum value of the index of refraction of the glass can be inferred from this information?

Answer: 1.22

Solution: Requiring total internal reflection at the point **B** implies

$$n\sin(\pi/2-\theta_1) = n\cos(\theta_1) = 1$$

and Snell's Law applied at the point **A** gives

$$n\sin(\theta_1) = \sin(\theta_0),$$

where **n** is the index of refraction of the glass and $\boldsymbol{\theta_0}$ is the incident angle. Squaring the first equation and using the second equation gives

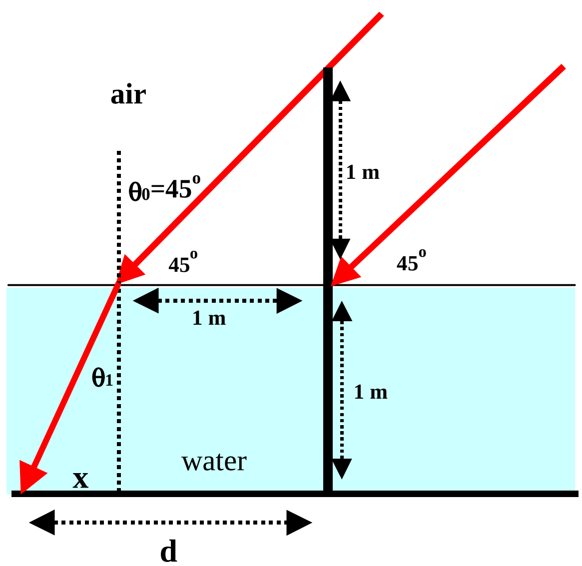
$$1 = n^2\cos^2(\theta_1) = n^2(1-\sin^2(\theta_1)) = n^2 - \sin^2(\theta_0).$$

Thus,

$$n = \sqrt{1+\sin^2(\theta_0)} = \sqrt{1+0.5} = 1.22,$$

where I used $\sin^2(\theta_0) = \sin^2(45^\circ) = 1/2$.

Problem 18:



A fiber optic cable (**n = 1.50**) is submerged in water (**n = 1.33**). What is the critical angle (**in degrees**) for light rays to stay inside the cable?

Answer: 62

Solution: The critical angle (angle of total internal reflection) is given by

$$\sin(\theta_c) = \frac{n_2}{n_1} = \frac{1.33}{1.5} = 0.887,$$

and thus **θ = 62.46^o**.

Problem 19:

A **2.0 meter** long vertical pole extends from the bottom of a swimming pool to a point **1.0 meter** above the surface of the water (**n = 1.33**) as shown in the **Figure**. If sunlight is incident **45 degrees** above the horizon, what is the length of the shadow of the pole (**in m**) on the level bottom of the pool?

Answer: 1.63

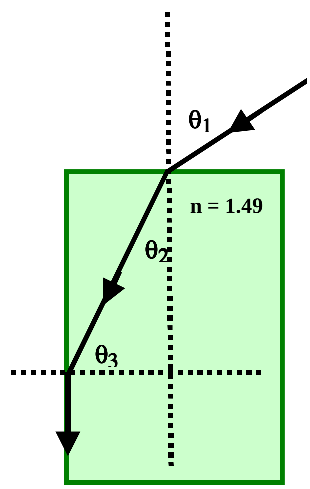
Solution: From Snell's law we have

$$\sin(\theta_1) = \frac{1}{n}\sin(\theta_0) = \frac{\sin(45^\circ)}{1.33},$$

and hence **θ_1 = 32^o**. Now the length of the shadow, **d**, is given by

$$d = 1m + x = 1m + (1m)\tan\theta_1 = 1m + (1m)\tan(32^\circ)$$
$$= 1m + 0.63m = 1.63m$$

Problem 20:



Light of wavelength **589 nm** is incident at an angle theta on the top surface of a polystyrene block (**n = 1.49**) as shown in the **Figure**. What is the maximum value of **θ** (**in degrees**) for which the refracted ray will undergo total internal reflection at the left vertical face of the block?

Answer: 90

Solution: From the Figure we see that

$$\theta_3 = \frac{\pi}{2} - \theta_2 \quad \text{and} \quad \sin(\theta_{3c}) = \frac{1}{1.49} = 0.671 .$$

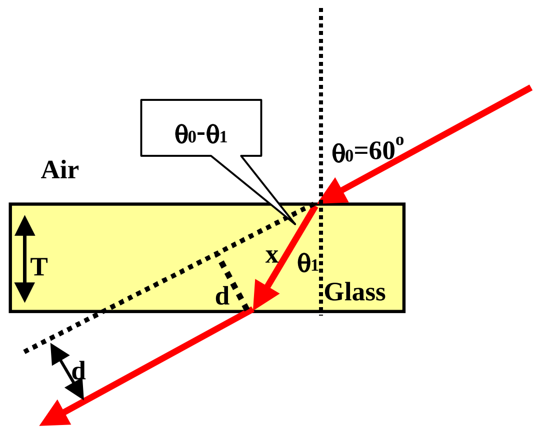
Thus, the critical angle for θ_3 is θ_{3c} = 42.15^o. We also know that

$$\sin(\theta_2) = \frac{1}{1.49}\sin(\theta_1),$$

which for θ_1 = 90^o yields θ_2 = 42.15^o which implies that θ_3 = 47.84^o > θ_{3c} which means the maximum angle for θ_1 is 90^o (any ray that enters the block with θ_1 > 0 will be totally internally reflected at the left face).

Problem 21:

A light ray enters a rectangular glass slab at an incident angle of **60 degrees**. The glass slab has an with index of refraction **n = 1.6** and a thickness **T**. The beam emerging from the other side of the glass slab is parallel to the incident beam but displaced by a distance **d** as shown in the **Figure**. What is the displacement **d**?



Answer: 0.54T

Solution: From **Snell's Law** we know that

$$n_1 \sin\theta_1 = n_0 \sin\theta_0$$

and solving for $\boldsymbol{\theta_1}$ gives

$$\sin\theta_1 = \frac{n_0 \sin\theta_0}{n_1} = \frac{\sin 60^\circ}{1.6} \quad \text{or} \quad \theta_1 = 32.77^\circ .$$

Now if we let **x** be the distance the light ray travels within the glass we see that

$$\sin(\theta_0 - \theta_1) = \frac{d}{x} \quad \text{and} \quad \cos\theta_1 = \frac{T}{x} .$$

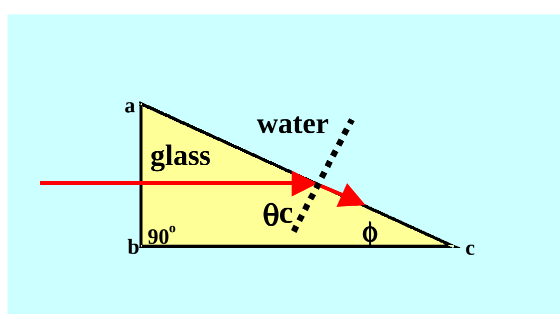
Solving for **d** gives

$$d = x\sin(\theta_0 - \theta_1) = \frac{\sin(\theta_0 - \theta_1)}{\cos\theta_1} T$$

$$= \frac{\sin(60^\circ - 32.77^\circ)}{\cos(32.77^\circ)} T = 0.54T$$

Problem 22:

A ray of light is incident normally of face **ab** of a glass prism in the shape of a right triangle and with index of refraction **n = 1.7** as shown in the **Figure**. If the prism is immersed in



water (**n = 1.3**), what is the largest value of the angle ϕ (**in degrees**) so that the ray is totally reflected at face **ac**?

Answer: 40.1

Solution: Total internal reflection at the face **ac** implies

$$n_1 \sin\theta_c = n_0 \sin 90^\circ$$

and solving for the critical angle θ_c gives

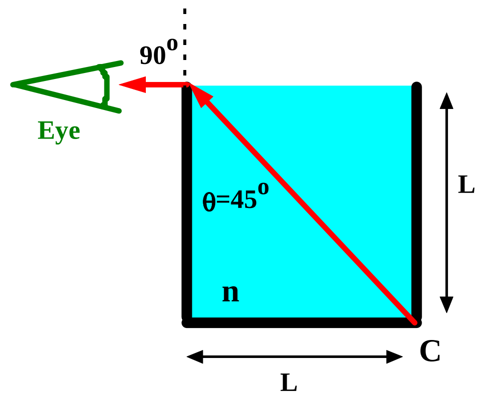
$$\sin\theta_c = \frac{n_0}{n_1} = \frac{1.3}{1.7} \quad \text{or} \quad \theta_c = 49.88^\circ .$$

We see that $\phi + \theta_c = 90^\circ$ and thus

$$\phi = 90^\circ - \theta_c = 90^\circ - 49.88^\circ = 40.12^\circ .$$

Problem 23:

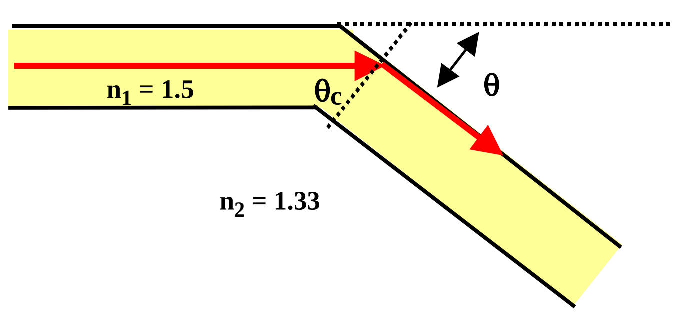
When the square metal tank with sides of length **L** in the Figure is filled to the top with an unknown liquid, an observer with eyes level with the top of the tank can just see the corner C. What is the index of refraction of the liquid?



Answer: 1.4

Solution: We see that

n sinθ = sin90° = 1 so that **n = 1/sinθ = 1/sin45° = 1.4**.



Problem 24:

A fiber optic cable (**n = 1.50**) is submerged in water (**n = 1.33**) as shown in the **Figure**. What is the maximum bending angle θ (**in degrees**) for light rays to stay inside the cable?

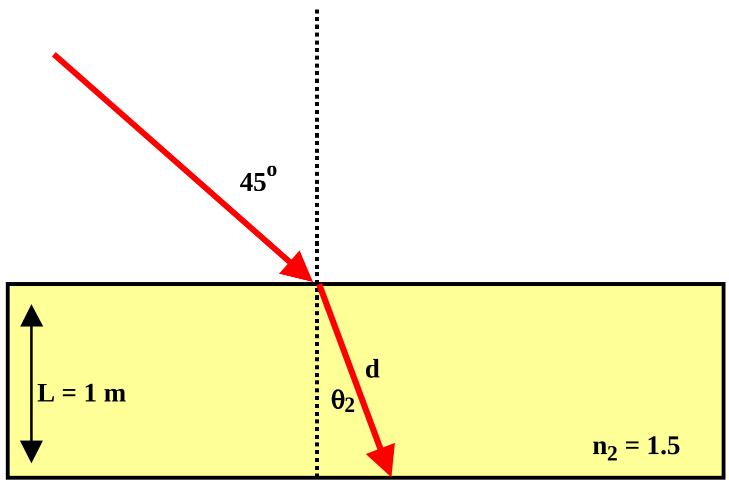
Answer: 27.5

Solution: The critical angle for total internal reflection is given by

$$\sin\theta_c = \frac{n_2}{n_1} = \frac{1.33}{1.5} = 0.887 \ ,$$

which implies that $\boldsymbol{\theta_c = 62.46^o}$. The maximum bending angle is thus,

$$\theta_{bend} = 90^\circ - \theta_c = 27.54^\circ \ .$$



Problem 25:

A beam of light strikes the surface of a **1-meter** thick parallel-sided sheet of glass (**n = 1.5**) at an incident angle of **45 degrees** as shown in the **Figure** . How long (**in nanoSeconds**) does it take the light to pass through the glass?

Answer: 5.7

Solution: The refracted angle $\boldsymbol{\theta_2}$ is given by

$$\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1 = \frac{1}{1.5}\sin 45^\circ = 0.471 \ ,$$

which implies $\boldsymbol{\theta_2 = 28.13^o}$. The distance, **d**, that the light must travel is given by

$$d = \frac{L}{\cos\theta_2} = \frac{1m}{\cos 28.13^\circ} = 1.134\, m \ ,$$

and the time is thus

$$t = \frac{d}{v} = \frac{nd}{c} = \frac{(1.5)(1.134\,m)}{3\times10^{8}\,m/s} = 5.67\,ns\ ,$$

where I used **n = c/v**.

Chapter 35 Solutions

Problem 1:

An object placed **180 cm** in front of a spherical mirror produces an image with a magnification **m = 0.1**. What happens to the image when the object is moved **150 cm** closer to the mirror?

Answer: the image increases in size by a factor of 4

Solution: We know that

$$\frac{1}{p}+\frac{1}{i}=\frac{1}{f} \qquad m=-\frac{i}{p},$$

and hence

$$f=\frac{p_1}{1-1/m_1}=\frac{180\ cm}{(1-10)}=-20\ cm,$$

where I have used $\mathbf{i = -m_1 p_1}$, and $\mathbf{m_1 = 0.1}$. Now, if I move the object **150 cm** closer to the mirror the new object position $\mathbf{p_2 = 30\ cm}$ (180 cm - 150 cm) and the new image position $\mathbf{i_2}$ will be given by

$$\frac{1}{i_2}=\frac{1}{f}-\frac{1}{p_2}=-\frac{1}{20\ cm}-\frac{1}{30\ cm}=-\frac{1}{12\ cm}.$$

Hence the new magnification is

$$m_2=\frac{-i_2}{p_2}=\frac{-(-12\ cm)}{30\ cm}=0.4,$$

which is **four times the original magnification**.

Problem 2:

A converging thin lens has a focal length of **30 cm**. For what ***two*** object locations (**in cm**) will this lens form an image **three** times the size of the object?

Answer: 20, 40

Solution: We know that

$$\frac{1}{p}+\frac{1}{i}=\frac{1}{f} \qquad m=-\frac{i}{p},$$

and hence

$$p = f\left(1 - 1/m\right).$$

For m = 3 we get

$$p = (30\,cm)\left(1 - 1/3\right) = \frac{2}{3}(30\,cm) = 20\,cm \quad ,$$

and for m = -3 we get

$$p = (30\,cm)\left(1 + 1/3\right) = \frac{4}{3}(30\,cm) = 40\,cm \quad .$$

Problem 3:

You decide to take a picture of the moon with your **35 mm** camera using its normal lens of **50 mm** focal length. Taking the moon's diameter to be approximately **$3.5\text{x}10^6$ m** and its distance from the earth **as $3.84\text{x}10^8$ m**, how large (**in mm**) is the moon's image on the film?

Answer: 0.46

Solution: We know that

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f} \qquad m = -\frac{i}{p} \, ,$$

and hence $i \approx f$ (because 1/p is very small) and since **$|m| = h_i/h_o$** we have

$$2h_i = |m|\, 2h_o = \frac{2h_o f}{p} = \frac{(3.5\times10^6\,m)(50\,mm)}{3.84\times10^8\,m} = 0.456\,mm \quad .$$

Problem 4:

Which of the following types of image of a real, erect object **cannot** be formed by a concave spherical mirror?

(A) Real, erect, enlarged
(B) Real, inverted, reduced
(C) Virtual. erect, enlarged
(D) Enlarged, inverted, and farther away than 2f
(E) Virtual, inverted, reduced?

Answer: A, E

Problem 5:

A concave mirror forms a real image which is twice the size of the object. If the object is **20 cm** from the mirror, the radius of curvature of the mirror (**in cm)** must be approximately:

Answer: 27

Solution: We know that

$$\frac{1}{p}+\frac{1}{i}=\frac{1}{f} \qquad m=-\frac{i}{p},$$

and hence

$$R=2f=\frac{2p}{1-1/m}=\frac{4p}{3}=26.7\,cm,$$

where I have used **i = -mp**, **f = R/2**, and **m = -2 (**since i is positive**)**.

Problem 6:

An erect image formed by a concave mirror (**f = 20 cm**) has a magnification of **2**. Which way and by how much should the object be moved to double the size of the image?

Answer: 5 cm further from the mirror

Solution: We know that

$$\frac{1}{p}+\frac{1}{i}=\frac{1}{f} \qquad m=-\frac{i}{p},$$

and hence

$$p=f\left(1-\frac{1}{m}\right),$$

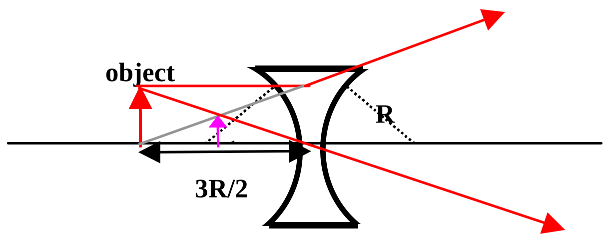
where I have used **i = -mp.** Thus,

$$p_1=20\,cm\left(1-\frac{1}{2}\right)=10\,cm, \text{ and}$$

$$p_2=20\,cm\left(1-\frac{1}{4}\right)=15\,cm.$$

We see that the object must be moved **5 cm = 15 cm –10 cm** further from the mirror.

Problem 7:



A diverging thin lens is constructed with index of refraction, **n**, and with radii of curvature of equal magnitude, **R**, as shown in the Figure. If an object is placed a distance **3R/2** from the lens forms a virtual image that one-half the size as the object, what is the index of refraction **n**?

Answer: 1.33

Solution: From the lensmakers formula we have

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (n-1)\left(-\frac{1}{R} - \frac{1}{R}\right) = -(n-1)\frac{2}{R}.$$

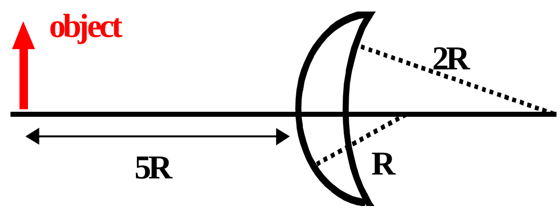
Also, we know that

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f} \qquad m = -\frac{i}{p}$$, and hence $$f = \frac{p}{\left(1 - \frac{1}{m}\right)},$$

where I have used **i = -mp**. Since **m = 1/2** we see that **f = -p = -3R/2**. Hence from the above equation,

$$n - 1 = -\frac{R}{2f} = \frac{1}{3}$$ and **n = 1.33**.

Problem 8:



A thin lens is constructed with index of refraction, **n = 1.5**, and with radii of curvature of equal to **R** and **2R** as shown in the Figure. What is the magnitude of the focal length and type of lens?

Answer: 4R, converging

Solution: The lensmakers formula tell us that

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = 0.5\left(\frac{1}{R} - \frac{1}{2R}\right) = \frac{1}{4R},$$

where I uses $\mathbf{r_1 = R}$ and $\mathbf{r_2 = 2R}$. Hence, **f = 4R** and the lens is converging since **f** is positive.

Problem 9:

What type of image is formed when an object is placed a distance **5R** from the lens in the previous problem?

Answer: Real inverted images 4 times the object height

Solution: We know that

$$\frac{1}{i} = \frac{1}{f} - \frac{1}{p} = \frac{1}{4R} - \frac{1}{5R} = \frac{1}{20R},$$

and hence i = 20R. The magnification is given by

$$m = \frac{-i}{p} = \frac{-20R}{5R} = -4 .$$

Thus the image is **real** (since i is positive) and **inverted** (since m is negative) and **4 times the object height** (since |m| = 4).

Problem 10:

Two objects placed **24 cm** apart in front of a converging thin lens both produce an image three times the size of the object. What is the focal length of the lens (**in cm**)?

Answer: 36

Solution: We know that

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f} \qquad m = -\frac{i}{p},$$

and hence

$$p = f(1 - 1/m).$$

For m = 3 we get

$$p_1 = f(1 - 1/3) = \frac{2}{3}f,$$

and for m = -3 we get

$$p_2 = f\left(1 + 1/3\right) = \frac{4}{3}f \ .$$

Subtracting the two equations yields

$$p_2 - p_1 = 24cm = \frac{4}{3}f - \frac{2}{3}f = \frac{2}{3}f \ ,$$

and thus **f = 3(24 cm)/2 = 36 cm**.

Problem 11:

A thin lens constructed with index of refraction, **n = 1.5**, produces an inverted real image that is twice the size of an object placed **1.5** meters from the lens. What happens to the image if the index of refraction of the lens is changed to **n = 1.25** (and the object remains 1.5 meters from the lens)?

Answer: becomes virtual, erect, and doubles in size

Solution: An inverted image twice the size of the object means **m = -2**, where

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f} \qquad m = -\frac{i}{p} \ ,$$

and hence

$$f_1 = f = \frac{p}{(1 - 1/m)} = \frac{1.5m}{(1 + 1/2)} = 1m \ .$$

We also know that

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right),$$

which implies that for fixed $\mathbf{r_1}$ and $\mathbf{r_2}$,

$$f_2 = \frac{(n_1 - 1)}{(n_2 - 1)} f_1 = \frac{(1.5 - 1)}{(1.25 - 1)} 1m = 2m \ .$$

The new image position given by

$$\frac{1}{i} = \frac{1}{f_2} - \frac{1}{p} = \frac{1}{2m} - \frac{1}{1.5m} = -\frac{1}{6m} \ ,$$

which means **i = -6 meters** and **m = -i/p = 4**. The new image is thus **virtual** (since **i** is now negative) and **erect** (since **m** is now positive) and **twice the size** (since **|m|** is now **4** instead of **2**).

Problem 12:

An object **6 cm** tall is located **10 cm** in front of a mirror. If the mirror produces a **3 cm-tall** upright image what kind of mirror is this and what is the magnitude of its radius of curvature?

Answer: convex, 20 cm

Solution: We know that

$$\frac{1}{p}+\frac{1}{i}=\frac{1}{f} \qquad m=-\frac{i}{p},$$

and hence

$$R=2f=\frac{2p}{1-1/m}=\frac{2p}{1-2}=-2p=-20\,cm,$$

where I have used **i = -mp**, **f = R/2**, and **m = 1/2**. Since **f < 0** the mirror is convex.

Problem 13:

A nearsighted person can see clearly only those objects which lie within **6 feet** of her eye. In order to see distant objects, she should wear eyeglasses of what type and focal length (**in ft**)?

Answer: diverging, 6

Solution: We know that

$$\frac{1}{p}+\frac{1}{i}=\frac{1}{f} \qquad m=-\frac{i}{p}.$$

When **p** is very large we get **f = i** and in this case we want **i = -6 feet** which means **f = -6 feet** (a diverging lens with |f| = 6 feet).

Problem 14:

A **2-cm-tall** object is placed **400 mm** from a converging lens, which is observed to form an image three times the size of the object. To make the image five times the size of the object, the object-lens distance (**in mm**) must be changed to:

Answer: 360

Solution: We know that

$$\frac{1}{p}+\frac{1}{i}=\frac{1}{f} \qquad m=-\frac{i}{p},$$

and hence

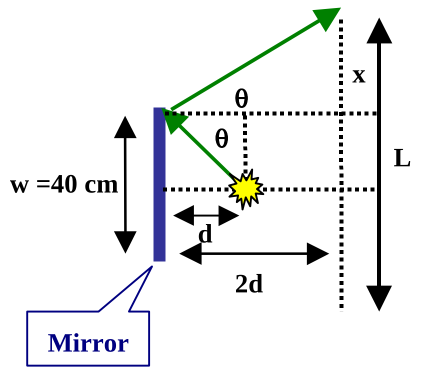
$$f=\frac{p_1}{1-1/m_1}=\frac{3p_1}{4},$$

where I have used $\mathbf{i = -m_1p_1}$, and $\mathbf{m_1 = -3}$ (since i is positive).
Now, from above I know that

$$p_2=f(1-1/m_2)=\frac{3p_1}{4}(1+1/5)=\frac{9p_1}{10}=360\ mm,$$

where I have used $\mathbf{i = -m_2p_2}$, and $\mathbf{m_2 = -5}$ (since i is positive).

Problem 15:



A light bulb burns in front of the center of a **40-cm** wide mirror that is hung vertically on a wall as shown in the **Figure**. A man walks in front of the mirror along a line that is parallel to the mirror and twice as far from it as the bulb. The greatest distance he can walk (**in cm**) and still see the image of the bulb in the mirror is:

Answer: 120

Solution: From the Figure we see that

$$\tan\theta=\frac{w/2}{d}=\frac{x}{2d},$$

which implies $\mathbf{x = w}$. The total path is thus, $\mathbf{L = 2x+w = 3w = 120cm}$.

Problem 16:

An object placed in front of a spherical mirror with focal length **f = -20 cm** produces an image with a magnification **m = 0.1**. If the the object is placed the same distance in front of a spherical mirror with focal length **f = -45 cm**, what happens to the image?

Answer: the image increases in size by a factor of 2

Solution: We know that

$$\frac{1}{p}+\frac{1}{i}=\frac{1}{f} \qquad m=-\frac{i}{p},$$

and hence

$$p = f(1-1/m).$$

For **m = 0.1** we get

$$p = f(1-10) = -9f = -9(-20\,cm) = 180\,cm.$$

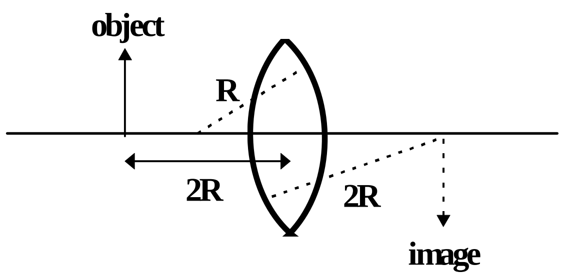
The new magnification is given by

$$m = \frac{-f}{p-f} = \frac{-(-45\,cm)}{180\,cm-(-45\,cm)} = \frac{45}{225} = 0.2,$$

so that the image **increases in size by a factor of 2**.

Problem 17:

A converging thin lens is constructed with index of refraction, **n**, and with radii of curvature of equal to **R** and **2R** as shown in the **Figure**. If an object placed a distance **2R** from the lens forms a real image that has the same size as the object but is inverted, what is the index of refraction **n**?



Answer: 1.67

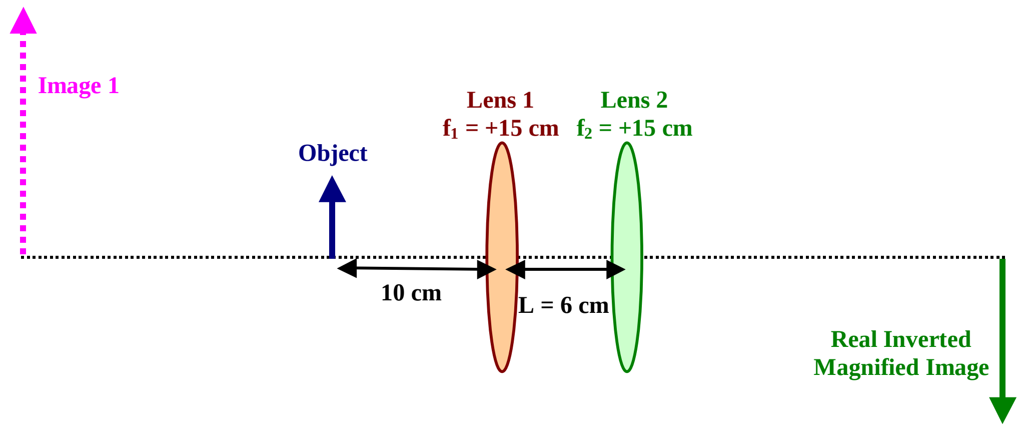
Solution: From the lensmakers formula we have

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) = (n-1)\left(\frac{1}{2R}+\frac{1}{R}\right) = (n-1)\frac{3}{2R}.$$

Also, since **m=-1** we know that **i= p** so that

$$\frac{2}{p}=\frac{2}{2R}=\frac{1}{f}$$, and thus **f = R**, and I have used **p = 2R**. Hence,

$$n-1 = \frac{2R}{3f} = \frac{2}{3}$$ and **n = 1.67**.



Problem 18:

Two identical converging lenses of focal lengths $\mathbf{f_1 = f_2 = +15}$ **cm** are separated by a distance **L = 6 cm**, as shown in the **Figure**. A luminous source is placed a distance of **10 cm** in front of the first lens. Locate the final image and determine the overall magnification of the system.

Answer: Image is real inverted and magnified (M = -2.14) and located 25.7 cm to the right of lens 2.

Solution: Ignoring lens 2 and taking $\mathbf{p_1 = 10}$ **cm** we get

$$\frac{1}{i_1} = \frac{1}{f_1} - \frac{1}{p_1} = \frac{1}{15\,cm} - \frac{1}{10\,cm} \quad \text{and} \quad i_1 = -30\,cm\ .$$

Also,

$$m_1 = -\frac{i_1}{p_1} = -\frac{-30\,cm}{10\,cm} = +3\ .$$

Thus, lens 1 produces a virtual non-inverted magnified image, which is to the **left** of lens 2. Now ignoring lens 1 and taking $\mathbf{p_2 = +(30cm+6cm) = +36}$ **cm** we get

$$\frac{1}{i_2} = \frac{1}{f_2} - \frac{1}{p_2} = \frac{1}{15\,cm} - \frac{1}{36\,cm} \quad \text{and} \quad i_2 = +25.71\,cm\ .$$

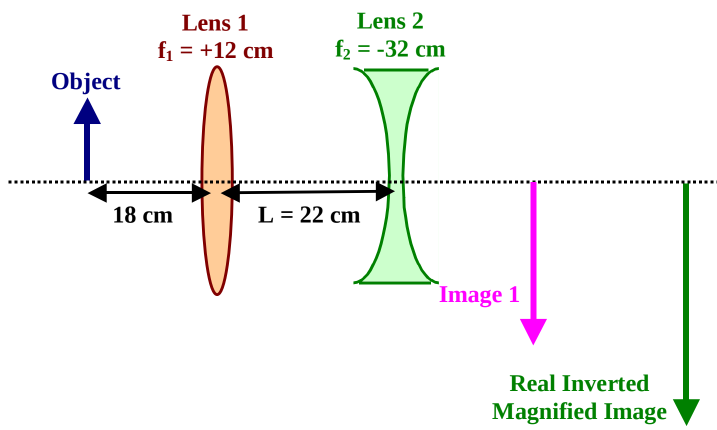
Also,

$$m_2 = -\frac{i_2}{p_2} = -\frac{25.71\,cm}{36\,cm} = -0.71$$

and

$$M = m_1 m_2 = (+3)(-0.71) = -2.14\ .$$

Thus, the system produces a **real inverted magnified image**, which is located **25.71 cm** to the right of lens 2.



Problem 19:

The optical system shown in the **Figure** consists of two lenses, of focal lengths $\mathbf{f_1 = +12\ cm}$ and $\mathbf{f_2 = -32\ cm}$, separated by a distance **L = 22 cm**. A luminous object with a height of **2.4 cm** is placed **18 cm** in front of the first lens. Find the position and the height of the final image.

Answer: Image is real inverted and magnified (M = -3.55) with a height of 8.53 cm and located 24.9 cm to the right of lens 2.

Solution: Ignoring lens 2 and taking $\mathbf{p_1 = 18\ cm}$ we get

$$\frac{1}{i_1} = \frac{1}{f_1} - \frac{1}{p_1} = \frac{1}{12\,cm} - \frac{1}{18\,cm} \quad \text{and} \quad i_1 = +36\,cm\ .$$

Also,

$$m_1 = -\frac{i_1}{p_1} = -\frac{36\,cm}{18\,cm} = -2\ .$$

Thus, lens 1 produces a real inverted magnified image, which is to the **right** of lens 2. Now ignoring lens 1 and taking $\mathbf{p_2 = -(36cm-22cm) = -14\ cm}$ we get

$$\frac{1}{i_2} = \frac{1}{f_2} - \frac{1}{p_2} = \frac{1}{-32\,cm} - \frac{1}{-14\,cm} \quad \text{and} \quad i_2 = +24.89\,cm\ .$$

Also,

$$m_2 = -\frac{i_2}{p_2} = -\frac{24.89\,cm}{-14\,cm} = +1.78$$

and

$$M = m_1 m_2 = (-2)(+1.78) = -3.55\ .$$

Thus, the system produces a **real inverted magnified image 24.89 cm** to the right of lens 2 with **height** = (3.55)(2.4cm) = **8.53 cm**.

Chapter 36 Solutions

Problem 1:

Light from a small region of a **100 Watt** incandescent bulb passes through a yellow filter and then serves as the source for a Young's double-slit interference experiment. Which of the following changes would cause the interference pattern to be more closely spaced?

(a) Use a blue filter instead of a yellow filter
(b) Use a 10 Watt bulb
(c) Use a 500 Watt bulb
(d) Move the bulb closer to the slits
(e) Move the slits closer together

Answer: (a) Use a blue filter instead of a yellow filter

Solution: The angular position of the bright fringes is given by

$$\sin\theta = \frac{m\lambda}{d}.$$

A more closely spaced pattern implies that for a given **m, θ** is smaller. Thus, we want to increase **d** or decrease λ and from **problem 15** we see that $\lambda_{blue} < \lambda_{yellow}$.

Problem 2:

The characteristic yellow light of sodium lamps arises from two prominent wavelengths in its spectrum, at approximately **589.0 nm** and **589.6 nm**, respectively. The light passes through a double slit and falls on a screen **10 m** away. If the slits are separated by a distance of **0.01 mm**, how far apart are the two second-order bright fringes on the screen (**in mm**)?

Answer: 1.2

Solution: The position of the second order bright fringe (**constructive interference**) is given by

$$\sin\theta \approx \theta = m\frac{\lambda}{d} = \frac{2\lambda}{d},$$

where I have set **m = 2** and used the small angle approximation. We also know that

$$\tan\theta \approx \theta = \frac{y}{L}.$$

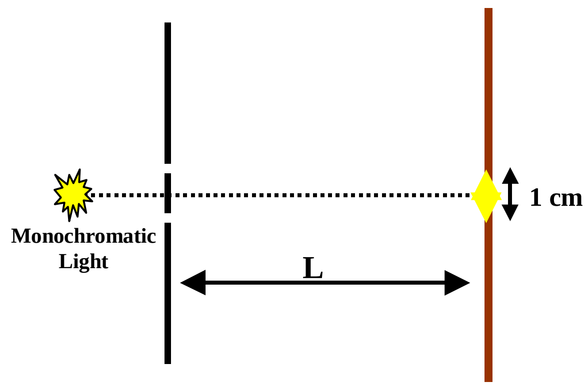
The position of the second order bright fringe is thus

$$y = \frac{2\lambda}{d}L,$$

and

$$y_2 - y_1 = 2(\lambda_2 - \lambda_1)\frac{L}{d} = \frac{2(0.6nm)(10m)}{10^{-5}m} = 1.2mm.$$

Problem 3:



A monochromatic light placed equal distance from two slits a distance **d** apart produces a central bright spot with a width of **1 cm** on a screen located a distance **L** away, as shown in the **figure**. If the entire apparatus is immersed in a clear liquid with index of refraction **n** the width of the central bright spot shrinks to **0.75 cm**. What is the index of refraction **n** of the clear liquid? (Note: assume L >> d)

Answer: 1.33

Solution: We know that for double-slit interference the position of the **first dark fringe** is given by

$$\sin\theta \approx \theta = \frac{\lambda}{2d}$$

$$\tan\theta \approx \theta = \frac{y}{L}$$

and hence the width, **w**, of the central bright spot is

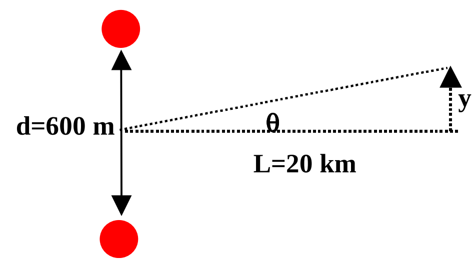
$$w = 2y = \frac{\lambda L}{d} \quad \text{and} \quad \frac{w_2}{w_1} = \frac{\lambda_2}{\lambda_1} = \frac{\lambda_0}{n\lambda_0} = \frac{1}{n},$$

where I used $\lambda_1 = \lambda_0$ and $\lambda_2 = \lambda_0/\mathbf{n}$ where λ_0 is the vacuum wavelength. Solving for **n** gives

$$n = \frac{w_1}{w_2} = \frac{1cm}{0.75\,cm} = 1.33\ .$$

Problem 4:

Two radio antennas are **600 m** apart along a north-south line and broadcast in-phase signals at frequency **1.0 MHz**. A receiver placed **20 km** to the east, equidistant from both antennas, picks up an acceptable signal (see Figure). How far due north of the present location (**in km**) would the receiver again detect a signal of nearly the same intensity?



Answer: 10

Solution: This is the same as **"two slit interference"** and the condition for **constructive interference** is

$$\sin\theta \approx \theta = m\frac{\lambda}{d} = \frac{\lambda}{d}\ ,$$

where I have set **m = 1** and used the small angle approximation. We also know that

$$\tan\theta \approx \theta = \frac{y}{L}\ .$$

Thus,

$$y = \frac{\lambda}{d}L = \frac{cL}{fd} = \frac{(3\times10^{8}\,m/s)(20\,km)}{(10^{6}\,/s)(600\,m)} = 10\,km\ ,$$

where I used **λ = c/f**.

Problem 5:

Monochromatic light is incident on a two slits **0.2 mm** apart. If the first order bright fringe is **4.8 mm** from the central bright spot on a screen located a distance of **1.5 m** from the double slit, what is the wavelength of the light (**in nm**)?

Answer: 640

Solution: We know that for double-slit interference

$$\sin\theta \approx \theta = \frac{m\lambda}{d}$$

$$\tan\theta \approx \theta = \frac{y}{L}$$

and hence

$$\lambda = \frac{yd}{mL} = \frac{(4.8\times10^{-3}m)(0.2\times10^{-3}m)}{1.5m} = 640\,nm\ ,$$

where I used **m = 1** for the 1st order bright fringe.

Problem 6:

A soap (water!) film has refractive index **1.34** and is **550 nm** thick. What wavelengths of visible light (**in nm**) are **not** reflected from it when it is illuminated from directly above by sunlight?

Answer: 491

Solution: Here we set the overall phase shift equal to the condition for maximum destructive interference as follows:

$$\Delta_{overall} = 2T + \lambda_{film}/2 = \left(m + \frac{1}{2}\right)\lambda_{film}\ ,$$

where $\lambda_{film} = \lambda_0/n_{soap}$. Solving for λ_0 gives

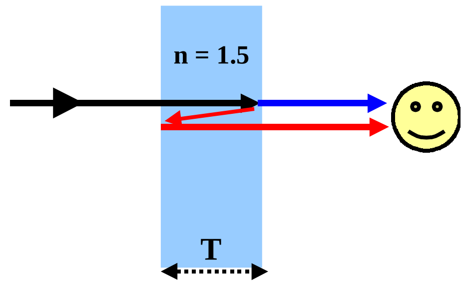
$$\lambda_0 = n_{soap}\frac{2T}{m} = \frac{1424\ nm}{m}\ ,$$

which produces the following table:

m	Wavelength λ_0
1	1424 nm
2	**737 nm**
3	**491 nm**
4	**369 nm**

with visible light having **400** < $\lambda_{visible}$ < **700 nm**.

Problem 7:



A beam of **600 nm** light is incident on the left side of a flat glass plate with refractive index **1.5** and part of it undergoes reflection between the faces of the plate as shown in the **Figure**. What minimum nonzero thickness **T** (**in nm**) of the glass plate will produce maximum brightness in the transmitted light?

Answer: 200

Solution: For **constructive interference** we set the overall phase shift equal to the condition for maximum constructive interference as follows,

$$\Delta_{overall} = 2T + 0 + 0 = m\lambda_{film} .$$

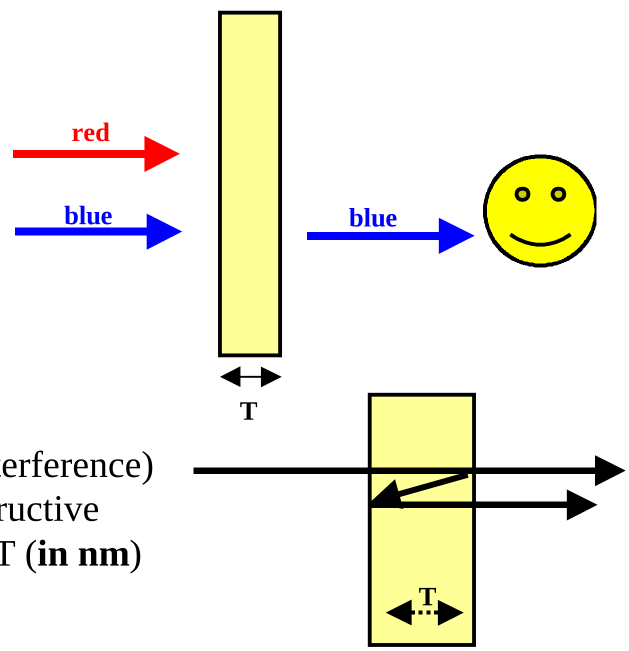
Solving for **T** gives

$$T = m\frac{\lambda_{film}}{2} = m\frac{\lambda_{\max}}{2n_{film}} .$$

The minimum thickness occurs when **m = 1** giving

$$T_{\min} = \frac{\lambda_{\max}}{2n_{film}} = \frac{600\ nm}{2(1.5)} = 200\ nm .$$

Problem 8:



A beam of red light (λ_{red} = **600 nm**) and a beam of blue light (λ_{blue} = **450 nm**) are incident on the left side of a flat glass plate with refractive index **1.5** as shown in the **Figure**. An observer on the other side of the glass plate sees a bright blue light (maximum constructive interference) and sees no red light (maximum destructive interference). What is the thickness T (**in nm**) of the glass plate?

Answer: 300

Solution: For the blue light and **constructive interference** we set the overall phase shift equal to the condition for maximum constructive interference as follows,

$$\Delta_{overall} = 2T + 0 + 0 = m\lambda_{film} .$$

Solving for **T** gives

$$T = m\frac{\lambda_{film}}{2} = m\frac{\lambda_{\max}}{2n_{film}} = m\frac{450\ nm}{2(1.5)} = m(150\ nm) .$$

For the constructive interference of the blue light we have the following possible thickness' of the glass:

m	T
1	150nm
2	**300nm**
3	450nm
4	600nm

For the red light and **destructive interference** we set the overall phase shift equal to the condition for maximum destructive interference as follows,

$$\Delta_{overall} = 2T + 0 + 0 = (m + 1/2)\lambda_{film} .$$

Solving for **T** gives

$$T = (m + 1/2)\frac{\lambda_{film}}{2} = (m + 1/2)\frac{\lambda_{\max}}{2n_{film}}$$

$$= (m + 1/2)\frac{600\ nm}{2(1.5)} = (m + 1/2)(200\ nm) .$$

For the destructive interference of the red light we have the following possible thickness' of the glass:

m	T
0	100nm
1	**300nm**
2	500nm
3	700nm

We see that **T = 300 nm** satisfies **both** conditions.

Problem 9:

A thin film of gasoline with a thickness of **400 nm** floats on a puddle of water. Sunlight falls almost perpendicularly on the film and reflects into your eyes. If $\mathbf{n_{gas} = 1.4}$ and $\mathbf{n_{water} = 1.33}$, which of the following wavelengths will be missing from the reflected beam due to destructive interference?

(a) 280 nm (b) 320 nm (c) 373 nm (d) 400 nm (e) 560 nm

Answer: ace

Solution: For **destructive interference** we set the overall phase shift equal to the condition for maximum destructive interference as follows,

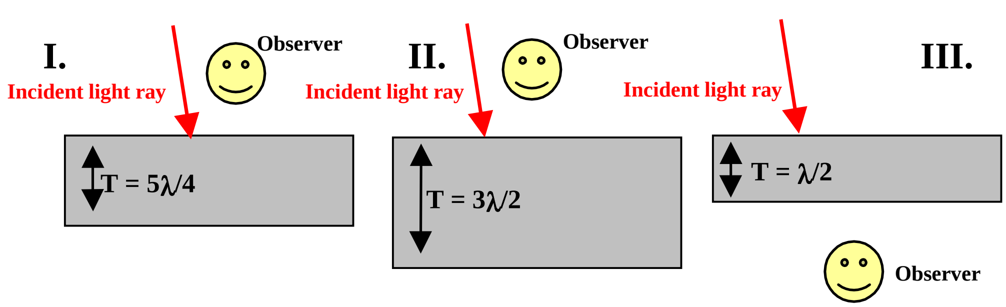
$$\Delta_{overall} = 2T + \lambda_{film}/2 + 0 = (m + 1/2)\lambda_{film} .$$

Solving for $\lambda = \lambda_{film}/n_{film}$ gives

$$\lambda = \frac{2Tn_{film}}{m} == \frac{2(400nm)(1.4)}{m} = \frac{1120nm}{m} .$$

Thus, the following wavelengths are missing due to destructive interference:

m	λ
1	1120nm
2	**560nm (e)**
3	**373nm (c)**
4	**280nm (a)**
5	224nm



Problem 10:

Three experiments involving a thin film (**in air**) are shown in the **Figure**. If **T** indicates the film thickness and **λ** is the wavelength of the light in the film, which experiments will produce **constructive** interference as seen by the observer?

Answer: I and III

Solution: For **I and II** setting the overall phase shift equal to the condition for maximum constructive interference gives:

$$\Delta_{overall} = 2T + \lambda_{film}/2 = m\lambda_{film} .$$

Solving for **T** gives

$$T = \left(m - \frac{1}{2}\right)\frac{\lambda_{film}}{2} ,$$

which produces the following table (where $\lambda = \lambda_{film}$):

I & II: Constructive	
m	**T**
1	$\lambda/4$
2	$3\lambda/4$
3	**$5\lambda/4$**
4	$7\lambda/4$

For **III** setting the overall phase shift equal to the condition for maximum constructive interference gives:

$$\Delta_{overall} = 2T = m\lambda_{film} .$$

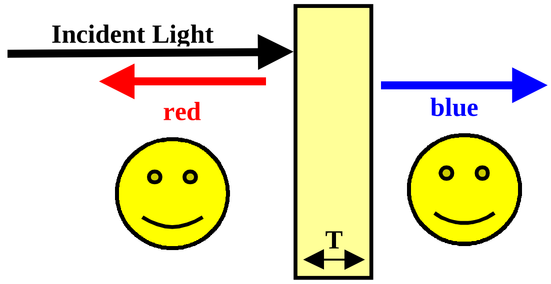
Solving for **T** gives

$$T = \frac{m\lambda_{film}}{2} ,$$

which produces the following table (where $\lambda = \lambda_{film}$):

III: Constructive	
m	**T**
1	**$\lambda/2$**
2	λ

Problem 11:



Two observers are on opposite sides of a flat glass plate with refractive index 1.5 as shown in the **Figure**. If the light is incident from the left and if the observer on the left sees maximum constructive interference for red light (**λ_{red} = 600 nm**) and the observer on the right sees maximum constructive interference for blue light (**λ_{blue} = 450 nm**), what is the thickness T (**in nm**) of the glass plate?

Answer: 300

Solution: For the observer on the **right** we have blue light and **constructive interference** so we set the overall phase shift equal to the condition for maximum constructive interference as follows,

$$\Delta_{overall} = 2T + 0 + 0 = m\lambda_{film} .$$

Solving for **T** gives

$$T = m\frac{\lambda_{film}}{2} = m\frac{\lambda_{\max}}{2n_{film}} = m\frac{450\,nm}{2(1.5)} = m(150\,nm) .$$

For the constructive interference of the blue light on the **right** we have the following possible thickness' of the glass:

m	T
1	150nm
2	**300nm**
3	450nm
4	600nm

For the observer on the **left** we have red light and **constructive interference** we set the overall phase shift equal to the condition for maximum constructive interference as follows,

$$\Delta_{overall} = 2T + \frac{\lambda_{film}}{2} + 0 = m\lambda_{film} .$$

Solving for **T** gives

$$T = (m - 1/2)\frac{\lambda_{film}}{2} = (m - 1/2)\frac{\lambda_{\max}}{2n_{film}}$$

$$= (m - 1/2)\frac{600\ nm}{2(1.5)} = (m - 1/2)(200\ nm).$$

For the constructive interference of the red light on the **left** we have the following possible thickness' of the glass:

m	T
1	100nm
2	**300nm**
3	500nm
4	700nm

We see that **T = 300 nm** satisfies **both** conditions.

Chapter 37 Solutions

Problem 1:

If monochromatic light of wavelength **500 nm** is incident on a single narrow slit **0.02 mm** wide, what is the width of the central bright spot (**in cm**) on a screen that is **3 meters** from the slit? (Hint: the width of the central bright spot is equal to the distance between the two first order minima.)

Answer: 15

Solution: The angular position of the first dark fringe is given by

$$\sin\theta \approx \theta = \frac{\lambda}{W}\text{, and } \tan\theta \approx \theta = \frac{y}{L}.$$

Thus, the width of the central bright spot, **D = 2y**, is given by

$$D = 2y = \frac{2\lambda}{W}L = \frac{2(500\times10^{-9}m)}{0.02\times10^{-3}m}(3m) = 15\,cm.$$

Problem 2:

Monochromatic light of wavelength **600 nm** is incident on a single narrow slit and produces a diffraction pattern on a screen **2 meters** away? If the screen is moved **1 meter** further away from the slit, what wavelength of monochromatic light (**in nm**) will produce a diffraction pattern with all the diffraction minima at the same points on the screen?

Answer: 400

Solution: The angular position of the dark fringes is given by

$$\sin\theta \approx \theta = \frac{m\lambda}{W}\text{, and } \tan\theta \approx \theta = \frac{y}{L}.$$

Solving for λ gives

$$\lambda = \frac{yW}{mL}$$

and for fixed **y**, **W**, and **m** we have

$$\frac{\lambda_2}{\lambda_1} = \frac{L_1}{L_2}$$

so that

$$\lambda_2 = \frac{L_1}{L_2}\lambda_1 = \frac{2m}{2m+1m}(600\,nm) = 400\,nm\ .$$

Problem 3:

Blue light (**λ = 470 nm**) falls on a wall with a single slit of width **0.18** mm and produces a diffraction pattern on a screen located a distance **L** from the slit. If the color of light is changed to red (**λ = 650 nm**), the width of the central bright spot on the screen increases by **1.0 cm**. What is the distance **L** to the screen (**in meters**)?

Answer: 5

Solution: The angular position of the first dark fringe is given by

$$\sin\theta \approx \theta = \frac{\lambda}{W}\text{ , and }\tan\theta \approx \theta = \frac{y}{L}\ .$$

Thus, the width of the central bright spot, **D = 2y**, is given by

$$D = 2y = \frac{2\lambda}{W}L\text{ , and }\Delta D = \frac{2\Delta\lambda}{W}L\ ,$$

where $\Delta D = D_2 - D_1$ and $\Delta\lambda = \lambda_2 - \lambda_1$. Solving for **L** gives

$$L = \frac{\Delta DW}{2\Delta\lambda} = \frac{(1\times 10^{-2}\,m)(0.18\times 10^{-3}\,m)}{2(180\times 10^{-9}\,m)} = 5m\ .$$